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Chapter 8

BINDINGS AND SPECTRA OF LIGHT NUCLEI WITH JISP16

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Abstract

We present here a comprehensive overview of *ab initio* results for binding energies and spectra of light nuclei obtained with the JISP16 NN interaction. The calculations were performed by means of the No-Core Full Configuration approach (NCFC) which extends the No-Core Shell Model (NCSM) to infinite basis space through extrapolation.

1. Introduction

An *ab initio* description of nuclear structure and reactions, i. e. high-precision simulations of many-nucleon systems based on realistic inter-nucleon interactions, is one of the mainstreams of modern nuclear theory [1]. It is based on a rapid development of supercomputer facilities and recent advances in the utilization of high-performance computing systems [2]. The modern *ab initio* nuclear theory has opened a wide range of nuclear phenomena that can be evaluated to high precision using realistic nucleon-nucleon (NN) and three-nucleon (NNN) interactions. In particular, *ab initio* approaches like the No-Core Shell Model (NCSM) [3], the Green's Function Monte Carlo (GFMC) [4] and the Coupled-Cluster Theory [5] are able to reproduce properties of a large number of atomic nuclei with mass up to $A = 16$ and a few heavier nuclear systems around closed shells. Very important progress has been achieved in the *ab initio* description of reactions with light nuclei, in particular, by combining the NCSM with the Resonating Group Method [6].

The *ab initio* theory requires a high-quality realistic strong interaction providing an accurate description of NN scattering data and predictions for binding energies, spectra and

other observables in light nuclei. A number of meson-exchange potentials sometimes supplemented with phenomenological terms to achieve high accuracy in fitting NN data (CD-Bonn [7], Nijmegen [8], Argonne [9]) have been developed that should be used together with modern NNN forces (Urbana[10, 11], Illinois [12], Tucson–Melbourne [13, 14, 15]) to reproduce properties of many-body nuclear systems. A very important step in the theory of inter-nucleon interactions in nuclei is the emergence of realistic NN and NNN interactions tied to QCD via Chiral Perturbation Theory [16, 17, 18, 19].

Three-nucleon forces require a significant increase of computational resources needed to diagonalize a many-body Hamiltonian matrix since the NNN interaction increases the number of non-zero matrix elements approximately by a factor of 30 in the case of p-shell nuclei [20, 21]. As a result, one needs to restrict the basis space in many-body calculations when NNN forces are involved that makes the predictions less precise. *Ab initio* many-body studies benefit from the use of recently developed purely two-nucleon interactions of INOY (Inside Nonlocal Outside Yukawa) [22, 23] and JISP (J-matrix Inverse Scattering Potential) [24, 25, 26, 27] types fitted not only to the NN data but also to binding energies of $A = 3$ and heavier nuclei. At the fundamental level, these NN interactions are supported by the work of Polyzou and Glöckle [28] who demonstrated that a realistic NN interaction is equivalent at the $A = 3$ level to some $NN + NNN$ interaction where the new NN force is related to the initial one through a phase-equivalent transformation (PET). It seems reasonable then to exploit this freedom and work to minimize the need for the explicit introduction of three- and higher-body forces. Endeavors along these lines have resulted in the design of INOY and JISP strong interaction models.

Conventional realistic meson-exchange NN interactions [7, 8, 9] and NN interactions obtained in Chiral Perturbation Theory [18] present convergence challenges in many-body calculations. To improve the convergence, these NN interactions are usually softened either using the Okubo–Lee–Suzuki (OLS) [29, 30] method reviewed in detail in Ref. [3] or the Similarity Renormalization Group (SRG) technique [31, 32]. Unfortunately, the convergence of many-body calculations with OLS-softened NN interactions is not monotonic as the basis space is enlarged, and hence the respective results cannot be extrapolated to the infinite basis space. The SRG softening of NN interaction induces NNN forces.

We present here the results for binding energies and energies of narrow (with width less than 300 keV) excited states of s- and p-shell nuclei based on NCSM calculations with the JISP16 [27, 33] NN interaction. The JISP interactions provide a fast convergence of NCSM calculations and are tuned with PETs to reduce the need for NNN forces. We extrapolate the NCSM results for energies to the infinite basis space following the No-Core Full Configuration (NCFC) approach [34]. The reliability of this approach was demonstrated by the prediction of the binding energy and spectra of the exotic proton-excess nucleus ^{14}F based on the JISP16 NN interaction [35]. The accuracy of the predictions was later confirmed by the first experimental observation of this nucleus [36]. Improved extrapolation techniques have been suggested recently [37, 38, 39, 40], and we use one of these techniques to obtain some of the results presented here.

After a brief description of the JISP16 NN interaction, we discuss the *ab initio* NCSM approach and various methods of extrapolation of the NCSM results for energies of ground and excited states to the infinite basis space (NCFC approach). Next we present the results for binding energies and spectra of narrow levels of s- and p-shell nuclei obtained with

JISP16 obtained up to now. We do not discuss results for both nuclei that are mirror pairs but present calculations only for the one with $Z < N$. The binding energies and spectra of some nuclei have been already published [21, 34, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49]. We present here all these results which are improved in some cases due to the use of larger basis spaces in NCSM calculations together with previously unpublished results. We compare our description of binding energies and spectra of light nuclei based on JISP16 with the GFMC description based on Argonne AV18 NN interaction supplemented by meson-exchange IL7 NNN force.

2. JISP16 NN Interaction

The J-matrix inverse scattering approach was introduced in Ref. [50]. It was further developed and used to design a high-quality JISP NN interaction in Ref. [24]. A nonlocal interaction obtained in this approach is in the form of a matrix in the oscillator basis in each of the NN partial waves. To reproduce scattering data in a wider energy range, one needs to increase the size of the interaction matrix and/or the $\hbar\Omega$ parameter of the oscillator basis. From the point of view of shell model applications, it is desirable however to reduce the size of interaction matrices and to use $\hbar\Omega$ values in the range of few tens of MeV. A compromise solution is to use $\hbar\Omega = 40$ MeV with $N_{\max} = 9$ truncation of interaction matrices, i. e., the JISP NN interaction matrices include all relative NN motion oscillator states with excitation quanta up to 8 or 9 depending on parity. In other words, we use interaction matrices of the rank $r = 5$ in s and p partial waves, $r = 4$ matrices in d and f partial waves, etc. In case of coupled waves, the rank of the interaction matrix is a sum of the respective ranks, e. g., the rank of the coupled sd-wave matrix is $r = 5 + 4 = 9$. The $N_{\max} = 9$ truncated JISP interaction with $\hbar\Omega = 40$ MeV provides an excellent description of np scattering data with $\chi^2/\mathrm{datum} = 1.03$ for the 1992 np data base (2514 data), and 1.05 for the 1999 np data base (3058 data) in the laboratory energy range up through 350 MeV [51]. The definition of the 1992 and 1999 np data bases is discussed in the review paper [52]. The NN data bases and analyses can be obtained also by the online SAID facility [53]. The JISP16 np phase shifts description is discussed in detail in Ref. [24]. The interaction is assumed to be charge-independent.

PETs originating from unitary transformations of the oscillator basis proposed in Refs. [54, 55], give rise to ambiguity of the interaction obtained in the J-matrix inverse scattering approach. This ambiguity is eliminated at the first stage by postulating the simplest tridiagonal form of the NN interaction in uncoupled and quasi-tridiagonal form in coupled NN partial waves [24]. At the next stage, PETs are used to fit the JISP interaction to various nuclear properties. First of all, the sd component of the NN interaction is modified with the help of PETs to reproduce the deuteron quadrupole moment Q and rms radius without violating the excellent description of scattering data. It is worth noting here that the deuteron binding energy E_d and asymptotic normalization constants are used as an input in the inverse scattering approach and are not affected by PETs.

Next, PETs were employed in other NN partial waves attempting to improve the description of binding energies and spectra of light nuclei in NCSM calculations. Following this *ab exitu* route, the JISP6 NN interaction fitted to properties of nuclei with masses $A \leq 6$, was proposed in Refs. [25, 26]. It was found out later that JISP6 strongly

overbinds nuclei with $A \geq 10$. Therefore a new fit of PET parameters was performed that resulted in the JISP16 NN interaction [27, 33] fitted to binding energies of nuclei with masses up through $A \leq 16$.

Generally, the JISP16 NN interaction provides a reasonable description of binding energies and spectra of nuclei with mass $A \leq 16$. However the fit of JISP16 to properties of light nuclei was performed in 2005–2006 [27] utilizing the OLS technique to improve the convergence of NCSM calculations. Since that time, new generations of supercomputers have become available which made it possible to perform NCSM calculations in larger basis spaces and the NCFC technique [34] was developed to include extrapolations with quantified uncertainties. It became clear [35, 41] that the OLS transformation does not produce a monotonic sequence in the binding energy at fixed oscillator energy which had been inferred from smaller space calculations. As we shall see, the new results obtained in larger basis spaces (without the OLS technique so the variational principle applies) and extrapolated to the infinite basis space clearly demonstrate that the JISP16 NN interaction is reasonably accurate in the description of spectra and binding energies of nuclei with $A \leq 12$ but provides overbinding of nuclei at the end of p shell.

3. No-Core Shell Model

We adopt here the NCSM with the NCFC extension discussed in the the next section for the *ab initio* modeling of light nuclei. The energies E_i and respective wave functions $\Psi_i(\vec{r}_1, ..., \vec{r}_A)$ of the ground and excited states of a nucleus with Z protons and N neutrons of mass $A = Z + N$ are obtained by solving an eigenproblem for the A-body Schrödinger equation

$$H\,\Psi_i(\vec{r}_1, ..., \vec{r}_A) = E_i\,\Psi_i(\vec{r}_1, ..., \vec{r}_A). \tag{1}$$

The wavefunction Ψ is expanded in an A-body basis of Slater determinants Φ_k of single-particle wavefunctions $\phi_{nljm}(\vec{r})$,

$$\Psi(\vec{r}_1, ..., \vec{r}_A) = \sum c_k\,\Phi_k(\vec{r}_1, ..., \vec{r}_A), \tag{2}$$

where

$$\Phi_k(\vec{r}_1, ..., \vec{r}_A) = \mathscr{A}[\phi_{n_1 l_1 j_1 m_1}(\vec{r}_1)\,\phi_{n_2 l_2 j_2 m_2}(\vec{r}_2)\,...\,\phi_{n_A l_A j_A m_A}(\vec{r}_A)], \tag{3}$$

and $\mathscr{A}$ is the antisymmetrization operator. Conventionally, harmonic oscillator wave functions are used as the single-particle basis functions $\phi_{nljm}(\vec{r})$, and we present below the results obtained with such a basis. However other single-particle basis functions can be also utilized in the NCSM calculations [56, 57]. All single-particle functions $\phi_{nljm}(\vec{r})$ are obtained with the same oscillator frequency $\hbar\Omega$ and are labelled by the quantum numbers n, l, j, and m, where n and l are the radial and orbital oscillator quantum numbers (with $N_i = 2n_i + l_i$ being the number of single-particle oscillator quanta), j is the total single-particle angular momentum, and m is its projection. The many-body basis states Φ_k have a well-defined parity and total angular momentum projection, $M = \sum m_i$, but they do not have a well-defined total angular momentum J. Thus, in two runs, one for each parity,

we can obtain the complete low-lying spectrum for a given value of the basis parameter $\hbar\Omega$, including the ground state, even if the spin of the ground state is not known a priori.

No model assumptions are used while constructing the many-body basis (3), it includes all possible many-body configurations. The only restriction that limits the accuracy of *ab initio* NCSM calculations is that the infinite-dimensional many-body basis (3) should be truncated in practical applications. Any finite truncation of the complete basis (3) produces NCSM results that due to the variational principle, provide a strict upperbound for the lowest state of a given spin and parity for a given nuclear potential. We are interested in the convergence with increasing basis space dimensions with the goal of obtaining, to within quantifiable uncertainties, by means of the NCFC extrapolations discussed below, the results corresponding to the complete basis. Following this route, we have to address eigenvalue problems for increasingly large sparse matrices with dimensions in some cases exceeding a billion. Improved algorithms to construct these matrices and to determine their lowest eigenstates, as well as efficient use of increasing computational resources, are critical for the success of this approach [58, 59, 60].

Here we use the so-called $N_{\max}$ truncation, which is a truncation on the total number of oscillator quanta of the many-body basis: the basis is limited to all many-body basis states with $\sum_i^A N_i \leq N_0 + N_{\max}$, where N_i is the number of quanta of each single-particle state in a many-body basis state, N_0 is the minimal number of quanta for the nucleus under consideration, and $N_{\max}$ is the truncation parameter. Note that for the many-body harmonic oscillator basis, this truncation makes it possible to factorize exactly the center-of-mass wavefunction and the relative wavefunction [61].

The many-body Hamiltonian H in Eq. (1) can be expressed as

$$H = H_A + \lambda\Big(H_{\rm CM} - \frac{3}{2}\hbar\Omega\Big), \tag{4}$$

where

$$H_A = T_{\rm rel} + \mathcal{V} = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk} + ... \tag{5}$$

is the Hamiltonian describing internal degrees of freedom of nucleus and the last term in Eq. (4) with a large enough parameter λ is responsible for shifting up in energy the A-body center-of-mass excited states. The net result is a set of low-lying eigenstates free from the spurious center-of-mass excitations (the so-called Lipkin–Lawson projection method [62]). In Eq. (5), m is the nucleon mass, p_i is the single-nucleon momentum operator, V_{ij} is the NN interaction including the Coulomb interaction between protons, V_{ijk} is the three-nucleon interaction, and we can allow for higher-body interactions as well if desired. However we primarily focus here on results obtained with the realistic two-body NN interaction JISP16 which does not require the use of NNN and higher-body interactions.

4. NCFC Extrapolations

The NCFC approach suggested in Ref. [34] adopted an empirical dependence

$$E(\hbar\Omega, N_{\max}) = E_\infty + A\, e^{-bN_{\max}} \tag{6}$$

of eigenenergies $E(\hbar\Omega, N_{\max})$ of some state obtained in NCSM calculations with the basis truncation $N_{\max}$ and a given value of $\hbar\Omega$. The fitting parameters A and b in Eq. (6) are functions of $\hbar\Omega$ and may be different for different ranges of $N_{\max}$ used to estimate E_∞, the energy of the state under consideration in the limit of infinite basis. Two extrapolation methods were proposed in Ref. [34]: a global extrapolation based on the results obtained in four successive basis spaces with five $\hbar\Omega$ values from a 10 MeV interval (Extrapolation A); and Extrapolation B based on the results obtained at various fixed $\hbar\Omega$ values in three successive basis spaces and defining the most reliable value for the extrapolation. These two extrapolations provided results consistent with each other and were carefully tested in a number of light nuclei where high-precision, essentially converged, result was achieved. A detailed description of these extrapolations and methods to evaluate their uncertainties illustrated by numerous examples can be found in Ref. [34]. We adopt here also an improvement in Extrapolation B proposed in Ref. [46] which leads in some cases to a larger and more realistic uncertainty estimate.

A detailed study of ultraviolet and infrared divergences of harmonic oscillator expansions [37] and further studies of Refs. [38, 39] resulted in the extrapolation formula

$$E(\hbar\Omega, N_{\max}) = E_\infty + A\, e^{-4\sqrt{\frac{E'}{\hbar\Omega}}\sqrt{N_0^{sp}+N_{\max}+i+\frac{3}{2}}}, \tag{7}$$

where $N_0^{sp} = 2n + l$ is the number of quanta of the highest single-particle oscillator state in the minimal possible many-body oscillator configuration of a given nucleus. According to the theory of Refs. [37, 38], E' should be equal to the energy in the infinite basis space limit E_∞ in a one-body or two-body problem. However setting $E' = E_\infty$ results in a poor quality of extrapolation and E' is conventionally used as an independent fitting parameter. The theory of Refs. [37, 38] suggests $i = 0$. Later it was found in Ref. [39] that a phenomenological value $i = 2$ better fits the data. The value $i = 2$ is consistent with the J-matrix formalism in scattering theory (see Ref. [63]) and we use it in our extrapolations. The extrapolation based on Eq. (7) is referred to below as Extrapolation C.

The extrapolation formula (7) is applicable when the ultraviolet cutoff parameter

$$\Lambda_{\rm UV} = \sqrt{\Big(N_0^{sp} + N_{\max} + \frac{3}{2}\Big) m\hbar\Omega} \tag{8}$$

takes large enough values [37, 38] depending on the NN interaction. According to recommendations of Ref. [37], in the case of JISP16 NN interaction, we can use the extrapolation (7) when

$$\Lambda_{\rm UV} \gtrsim 500 \text{ MeV}/c. \tag{9}$$

We however use here a stronger condition

$$\Lambda_{\rm UV} \gtrsim 600 \text{ MeV}/c. \tag{10}$$

We use the NCSM calculations of the ^{7}Li ground state with JISP16 NN interaction to illustrate in Fig. 1 how the extrapolation formula (7) works and how the Extrapolation C is designed using a least squares fitting strategy on data satisfying specified criteria. The results obtained by NCSM with various $\hbar\Omega$ values are shown by symbols (closed and open circles and diamonds), solid lines connect the results obtained with the same $N_{\max}$ which values are ranging from 2 through 14. Diamonds depict the results violating the inequality (10)

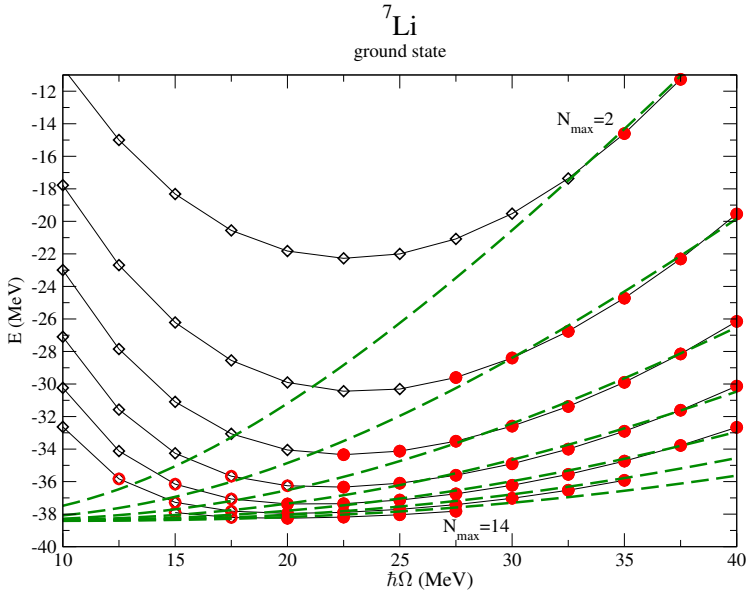

Figure 1. Ground state energy of ^{7}Li obtained in NCSM calculations with JISP16 NN interaction with various $\hbar\Omega$ values and $N_{\max}$ ranging from 2 to 14 in steps of 2: energies used in Extrapolation C are shown by closed circles; energies corresponding to $\Lambda_{\rm UV} < 600$ MeV/c and hence not used in Extrapolation C are shown by diamonds; open circles depict energies corresponding to $\Lambda_{\rm UV} \geq 600$ MeV/c but not used in the Extrapolation C since they are obtained with $\hbar\Omega$ values below the value associated with minimum of $E(\hbar\Omega, N_{\max})$ for the given $N_{\max}$. Dashed lines are calculations by Eq. (7) with parameters obtained by the fit.

which cannot be approximated by Eq. (7). The NCSM energies $E(\hbar\Omega, N_{\max})$ obtained with the same $N_{\max}$ have a minimum at some $\hbar\Omega$ value. On the other hand, Eq. (7) describes the energies $E(\hbar\Omega, N_{\max})$ monotonically increasing with $\hbar\Omega$ for a given $N_{\max}$. The NCSM results obtained with small enough values of $\hbar\Omega$ before the minimum of the function $E(\hbar\Omega, N_{\max})$ should be excluded from the fit of Extrapolation C. Those of these excluded results which still fit the inequality (10) are depicted by open circles in Fig. 1. Closed circles show the NCSM energies fitting the inequality (10) and increasing with $\hbar\Omega$ — only these results should be used in the fit by Eq. (7) to obtain values of parameters A, E' and E_∞ of the Extrapolation C. The dashed lines in Fig. 1 demonstrate calculations by formula (7) for various $N_{\max}$ using the least squares fitted values of the parameters.

Most of the energies E_∞ presented below obtained by Extrapolation C were calculated with the same prescription: we use for the fit all NCSM energies at the minimum of their $\hbar\Omega$ dependance and larger $\hbar\Omega$ values if they fit inequality (10). In addition, we exclude from the fit NCSM results obtained with small $N_{\max}$ values, i. e., results obtained with $N_{\max} < 4$ for positive and $N_{\max} < 3$ for negative parities.

According to the theory of Refs. [37], the ultraviolet cutoff (9) is the only restriction for the use of approximation (7) for the NCSM eigenenergies $E(\hbar\Omega, N_{\max})$. We use a stronger condition (10), however, the open circles in Fig. 1 clearly demonstrate that this condition is

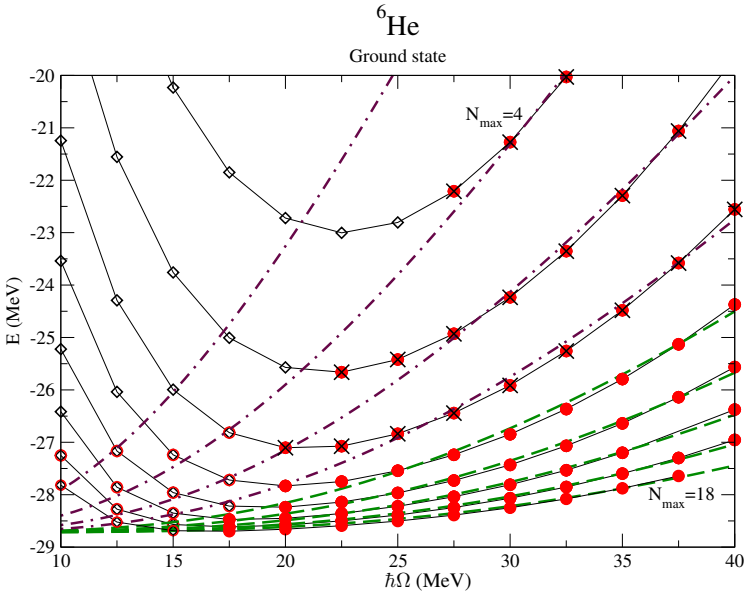

Figure 2. Ground state energy of ^{6}He obtained in NCSM calculations with JISP16 NN interaction with various $\hbar\Omega$ values and $N_{\max}$ ranging from 2 to 16 in steps of 2. Eigenenergies marked by crosses were additionally excluded from the fit to obtain a reasonable prediction for E_∞, dash-dotted lines show approximation of these energies by Eq. (7) with parameters obtained by the fit. See Fig. 1 for other details.

insufficient and should be supplemented by additional restrictions. We additionally exclude the eigenenergies $E(\hbar\Omega, N_{\max})$ before the minimum of their $\hbar\Omega$ dependence, however the need for even stronger restrictions become more obvious when the NCSM results obtained with larger $N_{\max}$ are available. The Extrapolation C is seen to reproduce well the NCSM results at sufficiently large $\hbar\Omega$ values. However, it fails around minima of $\hbar\Omega$ dependences for larger $N_{\max}$ where the NCSM eigenenergies lie above the Extrapolation C predictions as seen from Fig. 1. As a result, the Extrapolation C tends to underbind a nucleus predicting sometimes the E_∞ values above the strict variational upperbounds obtained in NCSM calculations. To obtain the Extrapolation C predictions below the variational upperbound, we used additional restrictions for a few levels in some light nuclei where the NCSM results obtained with large $N_{\max}$ values where available. For example, in the case of the ^{6}He ground state, we additionally excluded from the fit all states with $N_{\max} < 10$ shown by crosses in Fig. 2. It is seen from Fig. 2 that Extrapolation C reproduces the NCSM eigenenergies obtained with $N_{\max} < 10$ not included in the fit which lie outside the vicinity of the minima. We note however, that inclusion of the NCSM results obtained with small $N_{\max}$ values generally stabilizes the parameter fit of the Extrapolation C especially in cases when the results with large $N_{\max}$ values are not available: note, the vicinities of the minima are excluded for small $N_{\max}$ by the inequality (10). Clearly, the theory of Refs. [37] needs further development to formulate more adequate restrictions for the selection of NCSM eigenstates.

It is very important to estimate the uncertainties of Extrapolation C. It seems natural

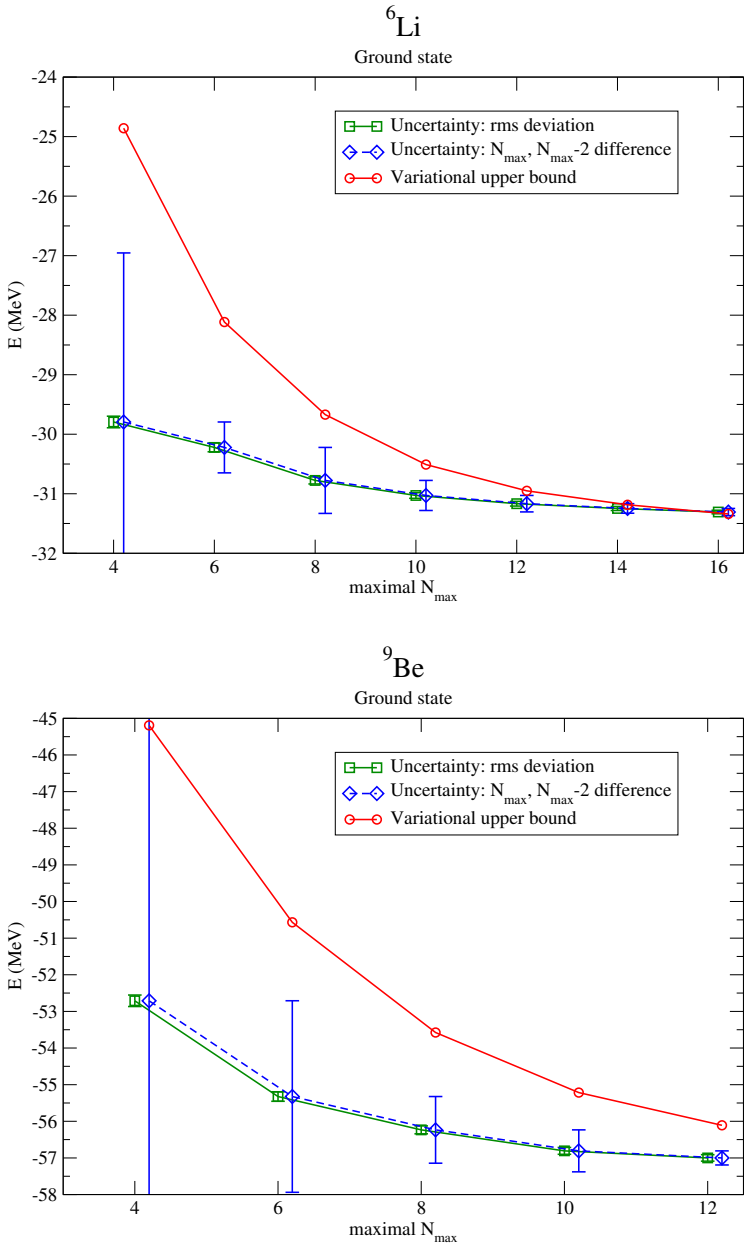

Figure 3. Variational upper bound (circles) and Extrapolation C for the ground state energies of ^{6}Li (upper panel) and ^{9}Be (lower panel): squares — uncertainties evaluated as a rms deviation of the NCSM results from calculations by Eq. (7) with parameters obtained by the fit; diamonds — uncertainties evaluated as a difference of E_∞ values from the fits to NCSM results obtained with all $N_{\max} \leq$ maximal $N_{\max}$ and with all $N_{\max} \leq$ maximal $N_{\max} - 2$.

to evaluate this uncertainty as a rms difference between the NCSM results and the values of the energy calculated with the same $\hbar\Omega$ and $N_{\max}$ by means of Eq. (7) with parameters obtained by the fit. However this idea does not work. The extrapolation formula (7) accurately reproduces energies acceptable for the fit and the suggested method is found in our test applications to underestimate the uncertainty of the extrapolation. Specifically, if we first calculate E_∞ including in the fit all acceptable NCSM results for a given nucleus obtained with all $N_{\max}$ less or equal to some maximal $N_{\max}$ value $N_{\max}^M$ and next add the NCSM results obtained with $N_{\max} = N_{\max}^M + 2$ to obtain E_∞ by a new fit, we see that the difference between these two E_∞ values is essentially larger than the above estimate of uncertainty (see Fig. 3). In other words, this method of evaluation of extrapolation uncertainty is inconsistent with results obtained by increasing the NCSM basis space.

Therefore we suggest another approach to evaluation of extrapolation uncertainty that is close to the uncertainty evaluation in Extrapolation B. We obtain E_∞ in a fit using all acceptable NCSM results obtained with all $N_{\max} \leq N_{\max}^M$. Next we exclude from the fit the results obtained with $N_{\max} = N_{\max}^M$ to obtain a new value of E_∞. The difference between these two E_∞ values is our estimate for the extrapolation uncertainty. In this case, we get a realistic uncertainty estimate consistent with increasing the NCSM basis space. This is illustrated by extrapolations of the ground state energies of ^{6}Li and ^{9}Be in Fig. 3.

A more elaborate extrapolation formula was suggested in Ref. [38]. In addition to the exponent involved in Eq. (7), it contains another exponent that should improve the description of the NCSM results obtained with smaller $\hbar\Omega$ values and is expected to describe the $\hbar\Omega$ dependance of $E(\hbar\Omega, N_{\max})$ before and after the minimum. Our numerous attempts to use this formula for extrapolating energies of a large enough set of light nuclei were unsuccessful. It appears that the two-exponent extrapolation formula of Ref. [38] is able to reproduce either the $\hbar\Omega$ dependence of $E(\hbar\Omega, N_{\max})$ at a given $N_{\max}$ or its $N_{\max}$ dependence at fixed $\hbar\Omega$ but fails to reproduce both these dependencies simultaneously.

5. Binding Energies and Spectra of Light Nuclei with JISP16 NN Interaction

We present in Tables I and II all available results for energies of states of s- and p-shell nuclei with the width not exceeding 300 keV — we do not attempt to describe here the broader resonances using an oscillator basis without coupling to the continuum. We however present the results only for one of each pair of mirror nuclei, the nucleus with $Z < N$ since mirror nuclei have similar properties. The tables list nearly all known s- and p-shell nuclei with $Z \leq N$ which have states with $\Gamma \leq 300$ keV. All experimentally known states with $\Gamma \leq 300$ keV are listed for each nucleus in order to also include the levels that we cannot describe. The only exception is ^{16}O which has many narrow excited states but we describe only its ground state. In some cases, our failure in description of a particular level is due to the limited supercomputer time — we need to perform more Lanczos iterations to obtain its converged energy in NCSM calculations at large $N_{\max}$ values. However there are levels whose description is not possible due to more fundamental reasons. A well-known example is the so-called Hoyle state, the first excited 0^+ state in ^{12}C. The Hoyle state was shown [70, 71] to have a strong contribution of many-body oscillator components with very

large $N_{\max}$ values which are impossible to include in calculations with modern computer facilities in the NCSM approach employing the $N_{\max}$ basis truncation. The impacts of the Hoyle state can be found in other nuclei. For example, we find states in ^{13}C which are formed by coupling of a single-neutron state to the Hoyle state in the ^{12}C cluster. Such states are, of course, also irreproducible in our calculations but may be eventually treated in a symmetry adapted basis such as demonstrated in Ref. [72].

The $N_{\max}^M$ column in the tables shows the maximal $N_{\max}$ values used in NCSM calculation. However, we sometimes present two $N_{\max}^M$ values, one of them in square brackets; the respective extrapolated results are also given in square brackets. In some cases, e. g., for ^{8}Be, we have a very limited set of results with different $\hbar\Omega$ with the largest $N_{\max}$ to perform Extrapolations A and C which is still adequate to can get a reasonable estimate of E_∞ by means of Extrapolation B, and we would like to present the Extrapolation B results using the largest NCSM calculations. In other cases, e. g., in ^{11}B we performed calculations with the largest $N_{\max}$ with the number of Lanczos iterations insufficient to get all excited states converged. We have the results for those states only with smaller $N_{\max}$ values. It seems reasonable to calculate excitation energies of such states using the results for the excited and ground states obtained in the same model space and hence to show these smaller basis space results for the ground state in the table.

5.1. Binding Energies

Binding energies are presented in the first row for each nucleus in Table I. The results obtained by Extrapolations A, B and C are consistent.

The uncertainties of Extrapolations A and B are known [34] to be approximately the same for a wide range of nuclei. We see that the uncertainty estimates of Extrapolation C for ground states in s and p shells are often similar to those of A and B. Extrapolation C has a theoretical foundation in contrast with the phenomenological Extrapolations A and B. However the Extrapolation C can be used only for large enough values of $\hbar\Omega$ where ground state energies obtained in NCSM increase with $\hbar\Omega$ and fails to reproduce the $\hbar\Omega$ dependance of eigenenergies $E(\hbar\Omega, N_{\max})$ in the vicinity of their minimum at fixed $N_{\max}$. The vicinity of the minimum of $E(\hbar\Omega, N_{\max})$ is important since the NCSM eigenenergies around this minimum are better approximations to E_∞. Extrapolations A and B describe the minimum of $E(\hbar\Omega, N_{\max})$, and this feature seems to compensate the lack of their theoretical grounding. As a result, the Extrapolation C does not seem to be especially advantageous as compared to Extrapolations A and B.

The JISP16 interaction correctly reproduces in most cases the experimental spin-parities of ground states. An exception is ^{11}Be. This nucleus demonstrates the so-called parity inversion: its ground state $\frac{1}{2}^+$ has an unnatural parity. In our calculations the natural parity state $\frac{1}{2}^-$ extrapolates below the $\frac{1}{2}^+$ state and appears to be the ground state. However the difference in our predictions for energies of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ is smaller than the uncertainty estimates. Therefore we cannot definitely state that JISP16 does not describe the parity inversion in ^{11}Be (see also Refs. [43, 46]). Experimentally these states are also found to have very close energies.

Table 1. Binding energies and spectra of natural parity states. All energies and widths are in MeV if other units are not indicated. The first line for each nucleus gives binding energies, all the remaining lines for this nucleus show excitation energies of excited states in the spectrum. The only exception is ^{11}Be whose ground state given in the first line is of unnatural parity, the next line gives the binding energy of the lowest state of natural parity and the remaining lines describe excitation energies relative to the lowest state of natural parity. The question mark in the second column indicates states whose isospin is not known (or doubtful) experimentally. The third column shows the maximal $N_{\max}$ value used in our calculations. Columns A, B and C show the results of respective extrapolations. The column Av. $\hbar\Omega$ shows excitation energies obtained by averaging excitation energies from the NCSM calculations with the largest available $N_{\max}$ value (indicated in the third column) over a set of reasonable $\hbar\Omega$ values. The column Av. $N_{\max}$ shows excitation energies obtained by averaging excitation energies from the NCSM calculations with $N_{\max} = N_{\max}^M$, $N_{\max} = N_{\max}^M - 2$, $N_{\max} = N_{\max}^M - 4$, $N_{\max} = N_{\max}^M - 6$. The AV18/IL7 column presents the results of GFMC calculations with Argonne AV18 NN and Illinois IL7 NNN interactions [64, 65, 66, 67, 68]. The last two columns present experimental data from [69]. Uncertainties of the last significant digit are given in round brackets after the value; see text for the method of evaluating uncertainties

	(J^π, T)	$N_{\max}^M$	A	B	C	Av. $\hbar\Omega$ Av. $N_{\max}$	AV18/IL7	Exp.	Width
^{3}H	$(\frac{1}{2}^+, \frac{1}{2})$	20	8.368(1)	8.369(2)	$8.368(^{+0.022}_{-0.001})$	–	8.47(0)	8.482	12.32 years
^{4}He	$(0^+, 0)$	18	28.299(4)	28.299(0)	28.300(0)	–	28.43(0)	28.296	stable
^{6}He	$(0^+, 1)$	18	28.80(5)	28.803(6)	28.706(3)	–	29.20(3)	29.269	807 ms
^{6}He	$(2^+, 1)$	18	2.2(1)	2.26(12)(6)	$2.39(^{+0.01}_{-0.03})$	2.55(4) 2.63(2)	–	1.80(3)	0.113
^{6}Li	$(1^+, 0)$	16	31.47(9)	31.49(6)	31.42(5)	–	31.93(3)	31.995	stable
^{6}Li	$(3^+, 0)$	16	2.56(4)	2.552(65)(5)	$2.60(^{+0.04}_{-0.05})$	2.550(5) 2.50(1)	2.34(3)	2.186(2)	0.024
^{6}Li	$(0^+, 1)$	16	3.8(1)	3.68(8)(3)	3.72(6)	3.75(2) 3.75(2)	3.50(3)	3.563	$8.2 \cdot 10^{-6}$
^{7}Li	$(\frac{3}{2}^-, \frac{1}{2})$	14	38.6(1)	38.6(1)	38.42(8)	–	39.0(1)	39.245	stable
^{7}Li	$(\frac{1}{2}^-, \frac{1}{2})$	14	0.5(2)	0.52(6)(2)	0.6(1)	0.60(1) 0.73(2)	0.1(1)	0.478	73 fs
^{7}Li	$(\frac{7}{2}^-, \frac{1}{2})$	14	5.2(2)	5.25(10)(2)	5.4(1)	5.288(7) 5.26(1)	4.9(1)	4.630(9)	0.093
^{7}Li	$(\frac{5}{2}^-, \frac{1}{2})$	14	7.1(4)	7.1(2)(1)	7.4(1)	7.45(6) 7.88(7)	6.5(1)	6.68(5)	0.88
^{7}Li	$(\frac{5}{2}^-, \frac{1}{2})$	14	8.0(5)	8.10(20)(5)	8.4(1)	8.36(5) 8.53(3)	7.7(2)	7.46(1)	0.089
^{7}Li	$(\frac{3}{2}^-, \frac{3}{2})$	14	12.2(2)	12.1(2)(1)	12.3(2)	11.60(4) 11.1(2)	–	11.24(3)	0.26
^{8}He	$(0^+, 2)$	14	29.9(2)	29.9(2)	29.4(2)	–	31.06(15)	31.409	119 ms
^{8}Li	$(2^+, 1)$	12	40.3(3)	40.3(3)	40.1(4)	–	41.5(2)	41.278	840 ms
^{8}Li	$(1^+, 1)$	12	1.5(4)	1.50(20)(5)	1.6(4)	1.71(5) 1.96(6)	1.4(2)	0.981	8.2 fs
^{8}Li	$(3^+, 1)$	12	2.8(3)	2.77(20)(1)	2.8(4)	2.752(7) 2.65(2)	3.0(3)	2.255(3)	0.032
^{8}Li	$(4^+, 1)$	12	7.1(5)	7.1(2)(1)	7.3(5)	7.59(8) 8.1(1)	6.7(3)	6.53(2)	0.035
^{8}Li	$(0^+, 2)$	–	–	–	–	–	–	10.822	<0.012
^{8}Be	$(0^+, 0)$	10 [12]	55.2(5)	55.35(3) [55.5(3)]	54.8(9)	–	56.3(1)	56.500	$6 \cdot 10^{-6}$
^{8}Be	$(2^+, 1?)$	10 [12]	16.8(5)	16.86(30)(4) [16.7(1)]	16.7(9)	16.66(1) 16.49(4)	16.8(2)	16.626(3)	0.108
^{8}Be	$(2^+, 0?)$	10 [12]	17.2(7)	17.1(5)(2) [17.3(5)]	17.4(9)	17.40(3) 17.47(2)	16.8(2)	16.922(3)	0.074
^{8}Be	$(1^+, 1?)$	10 [12]	18.3(7)	18.23(50)(7) [18.2(2)]	18.3(9)	18.60(7) 18.69(5)	17.5(2)	17.640(1)	0.0107
^{8}Be	$(1^+, 0)$	10 [12]	18.9(9)	18.7(7)(4) [18.8(3)]	19(2)	20.1(2) 20.9(2)	18.0(2)	18.150(4)	0.138
^{8}Be	$(3^+, 1?)$	10 [12]	19.5(5)	19.6(3)(1) [19.5(1)]	19.5(8)	19.33(3) 19.05(7)	19.4(2)	19.07(1)	0.271
^{8}Be	$(3^+, 0?)$	10 [12]	21(1)	20.0(9)(3) [19.8(2)]	21.1(8)	21.7(2) 22.0(3)	19.9(2)	19.24(1)	0.227
^{8}Be	$(0^+, 2)$	–	–	–	–	–	–	27.494	0.0055
^{9}Li	$(\frac{3}{2}^-, \frac{3}{2})$	12	43.6(3)	43.7(3)	43.4(4)	–	45.2(3)	45.342	178 ms
^{9}Be	$(\frac{3}{2}^-, \frac{1}{2})$	10 [12]	57.0(6)	57.09(5) [57.2(3)]	56.8(6) [57.0(2)]	–	58.1(2)	58.167	stable
^{9}Be	$(\frac{5}{2}^-, \frac{1}{2})$	10 [12]	2.82	2.9(4)(1) [2.8(1)]	2.9(6)	2.960(9) 3.12(3)	2.4(2)	2.429(1)	0.001
^{9}Be	$(\frac{3}{2}^-, \frac{3}{2})$	10	15.1(6)	15.26(33)(5)	15.4(7)	14.96(5) 14.5(1)	–	14.392(2)	$365 \cdot 10^{-6}$
^{10}Li	$(1^+, 2?)$	10	42(1)	42.0(2)	41(1)	–		45.11(4)	0.1
^{10}Be	$(0^+, 1)$	10	64.1(5)	64.0(4)	64.0(4)	–	64.1(3)	64.979	$1.5 \cdot 10^6$ years
^{10}Be	$(2^+, 1)$	10	3.5(6)	3.53(30)(4)	3.6(5)	3.69(2) 3.89(4)	3.4(3)	3.368	125 fs
^{10}Be	$(2^+, 1)$	10	6.2(9)	6.1(6)(2)	7(1)	7.2(1) 8.3(2)	5.3(3)	5.958	<55 fs
^{10}Be	$(0^+, 1)$	–	–	–	–	–	–	6.179	1 ps
^{10}Be	$(2^+, 1)$	10	9.5(7)	9.6(5)(1)	9.1(3)	10.13(5) 11.2(2)	–	7.542	0.006
^{10}Be	$(2^+, 1?)$	10	11(1)	10.8(9)(7)	12(2)	11.2(5) 10.5(4)	–	9.560	0.141

^{10}B	$(3^+,0)$	10 [8]	63.8(5) [63(1)]	63.7(3) [64.0(2)]	63.6(4) [63(2)]	–	64.8(9)	64.753	stable
^{10}B	$(1^+,0?)$	10	0.4(8)	0.9(2.4)(0.9)	1(1)	2.4(8) 5.3(7)	0.7(8)	0.718	0.707 ns
^{10}B	$(0^+,1?)$	8	2(1)	1.78(1.5)(0.03)	2(1)	2.3(2) 2.4(1)	2.2(2)	1.74	4.9 fs
^{10}B	$(1^+,0?)$	10	2(1)	4.1(9)(3)	2.9(9)	3.4(4) 4.5(4)	3.0(6)	2.154	1.48 ps
^{10}B	$(2^+,0?)$	10	3.4(8)	3.8(8)(4)	3.7(5)	4.30(6) 5.9(2)	3.9(5)	3.587	102 fs
^{10}B	$(3^+,0)$	10	6(1)	5.7(9)(5)	6.2(8)	7.3(2) 9.0(2)	–	4.774	$8 \cdot 10^{-6}$
^{10}B	$(2^+,1)$	8	6(1)	5.3(2.0)(0.3)	5(2)	6.1(1) 6.30(9)	5.8(3)	5.164(1)	$2 \cdot 10^{-6}$
^{10}B	$(1^+,0?)$	–	–	–	–	–	–	5.182(8)	0.11
^{10}B	$(2^+,0)$	10	5(1)	5.2(9)(5)	5(1)	7.0(6) 10.1(7)	–	5.920	0.006
^{10}B	$(4^+,0?)$	10	5.3(7)	5.7(4)(1)	5.5(6)	5.72(8) 6.5(1)	5.6(5)	6.025	0.00005
^{10}B	$(3^+,0)$	10	9(1)	9.5(9)(6)	9.1(7)	7.0(6) 10.1(7)	–	7.004(5)	0.098
^{10}B	$(2^+,?)$	–	–	–	–	–	–	7.470(4)	0.065
^{10}B	$(0^+,1)$	–	–	–	–	–	–	7.5599(4)	0.003
^{10}B	$(2^+,1?)$	8	8(3)	8.2(2.4)(0.5)	9(2)	10.3(3) 11.2(2)	–	8.895	0.039
^{11}Li	$(\frac{3}{2}^-,\frac{5}{2})$	8	42(3)	42.4(1)	40(2)	–		45.65(2)	8.75 ms
^{11}Be	$(\frac{1}{2}^+,\frac{3}{2})$	11 [9]	[62(3)]	63.3(8)	60(3)	–		65.483	13.8 s
^{11}Be	$(\frac{1}{2}^-,\frac{3}{2}?)$	10	64.0(9)	64.0(6)	63(1)	–		65.163	115 fs
^{11}Be	$(\frac{3}{2}^-,\frac{3}{2}?)$	10	3(1)	2.7(7)(2)	4(2)	3.36(7) 3.61(8)		2.33(1)	0.206
^{11}Be	$(\frac{3}{2}^-,\frac{3}{2}?)$	10	–	–	–	–		3.080(6)	0.122
^{11}Be	$(\frac{3}{2}^-,\frac{3}{2}?)$	10	–	–	–	–		3.635(1)	0.01
^{11}Be	$(\frac{5}{2}^-,\frac{3}{2}?)$	10	3.4(9)	3.53(40)(5)	3(1)	3.2(1) 3.1(1)		4.935(3)	0.45
^{11}Be	$(\frac{3}{2}^-,\frac{3}{2}?)$	10	–	–	–	–		7.70(2)	0.23
^{11}Be	$(\frac{3}{2}^-,\frac{3}{2}?)$	10	6(1)	10(3)(2)	9(2)	8.9(4) 10.4(4)		8.49(3)	0.2
^{11}Be	$(\frac{5}{2}^-,\frac{3}{2}?)$	10	9(1)	9.2(5)(2)	9(1)	9.69(2) 10.2(1)		10.27(5)	0.21
^{11}B	$(\frac{3}{2}^-,\frac{1}{2})$	10 [8]	76.0(4) [75(1)]	76.1(4) [76(1)]	76.1(8)[75(2)]	–		76.206	stable
^{11}B	$(\frac{1}{2}^-,\frac{1}{2}?)$	10	0.8(6)	0.7(4)(1)	1(1)	1.41(7) 2.5(2)		2.125	3.8 fs
^{11}B	$(\frac{5}{2}^-,\frac{1}{2}?)$	10	3.2(6)	3.2(4)(1)	2.9(7)	3.73(4) 4.6(2)		4.445	0.82 fs
^{11}B	$(\frac{3}{2}^-,\frac{1}{2}?)$	10	4.3(7)	4.3(4)(2)	4(1)	5.7(1) 7.4(3)		5.020	0.24 fs
^{11}B	$(\frac{7}{2}^-,\frac{1}{2}?)$	10	5.9(6)	5.9(4)(1)	6(1)	6.57(6) 7.6(2)		6.743(2)	15 fs
^{11}B	$(\frac{5}{2}^-,\frac{1}{2}?)$	8	10(2)	10.0(1.0)(0.4)	10(3)	11.8(2) 13.1(3)		8.920(2)	0.82 fs
^{11}B	$(\frac{3}{2}^-,\frac{1}{2}?)$	8	12(2)	12(4)(1)	13(3)	14.1(1) 15.7(4)		10.262(8)	0.163
^{11}B	$(\frac{5}{2}^-,\frac{1}{2}?)$	–	–	–	–	–		10.33(1)	0.112
^{11}B	$(\frac{5}{2}^-,\frac{1}{2}?)$	–	–	–	–	–		11.89(1)	0.194
^{11}B	$(\frac{1}{2}^-,\frac{1}{2}?)$	8	12(2)	11.3(2.8)(0.7)	12(3)	13(1) 14(1)		12.92(1)	0.23
^{12}Be	$(0^+,2)$	10 [8]	66.9(8) [66(4)]	67(1) [67(2)]	65.2(4) [64(1)]	–		68.651	21.3 ms
^{12}Be	$(2^+,2)$	8	3(3)	3.03(3.20)(0.02)	2(3)	2.78(9) 2.64(7)		2.102(12)	?
^{12}B	$(1^+,1)$	10 [8]	[79(2)]	80.0(5) [80(2)]	78.80(7) [79(3)]	–		79.577(2)	20.2 ms
^{12}B	$(2^+,1?)$	10 [8]	[-0.2(1.6)]	0.2(7)(3)	-0.3(4)	-1.0(2) -1.5(2)		0.953	180 fs
^{12}B	$(2^+,1?)$	10 [8]	[3(2)]	3.1(1.2)(0.4)	3.2(3)	3.9(1) 5.4(2)		3.759(6)	0.040
^{12}B	$(1^+,1?)$	8	4(2)	3.3(1.8)(0.5)	4(4)	5.32(8) 6.7(3)		5.00(2)	0.050
^{12}B	$(3^+,1?)$	8	6(2)	5.1(3.1)(0.7)	6(4)	7.5(1) 8.9(3)		5.612(8)	0.110
^{12}B	$(1^+,1?)$	8	7(3)	7.4(1.3)(0.7)	7(4)	11.3(4) 13.5(6)		6.600	0.140
^{12}B	$(0^+,2)$	6	–	17(3)(1)	18(5)	20.9(8) 21.9(7)		12.75(5)	0.085
^{12}C	$(0^+,0)$	10 [8]	94.9(4) [94.1]	94.8(4) [95(3)]	95.3(3) [95(2)]	–	93.3(4)	92.163	stable
^{12}C	$(2^+,0)$	10	3.8(5)	3.86(40)(6)	3.7(3)	4.19(3) 5.1(2)	3.9(9)	4.439	10^{-8}
^{12}C	$(0^+,0)$	–	–	–	–	–	10.4(5)	7.654	$8.5 \cdot 10^{-6}$
^{12}C	$(1^+,0)$	10	12.7(5)	12.76(60)(6)	12.7(4)	12.91(6) 14.0(3)		12.710(6)	$18 \cdot 10^{-6}$
^{12}C	$(4^+,0)$	10	13.5(8)	13.6(9)(2)	13.3(5)	14.95(9) 17.4(4)		14.083	0.258
^{12}C	$(1^+,1)$	10	17.8(7)	17.5(6)(3)	18(1)	19.2(2) 20.6(3)		15.110(3)	$44 \cdot 10^{-6}$
^{12}C	$(2^+,1)$	10	17(3)	18.6(1.0)(0.9)	17.8(8)	18.0(2) 19.2(2)		16.106(1)	0.005
^{12}C	$(0^+,1)$	10 [8]	[19(2)]	[18(5)(1)]	20(1)	20.9(6) 24.0(6)		17.76(2)	0.08
^{12}C	$(2^+,1)$	8	21(2)	20.3(2.9)(0.7)	$24(^{+3}_{-6})$	24.4(4) 26.2(5)		18.80(4)	0.1
^{12}C	$(1^+,?)$	–	–	–	–		–	– 19.69	0.23
^{12}C	$(0^+,2)$	–	–	–	–	–	–	27.595(24)	0.08
^{13}B	$(\frac{3}{2}^-,\frac{3}{2})$	8	85(2)	87(4)	86(3)	–		84.455(1)	17.33 ms
^{13}C	$(\frac{1}{2}^-,\frac{1}{2})$	8 [6]	101(1)	102(3) [98(5)]	102(3) [99(6)]	–		97.110	stable
^{13}C	$(\frac{3}{2}^-,\frac{1}{2}?)$	8	4(1)	3.3(5.2)(0.5)	4(4)	6.0(2) 7.5(3)		3.685	1.10 fs
^{13}C	$(\frac{5}{2}^-,\frac{1}{2}?)$	8	6(1)	5.5(5.2)(0.9)	9.5(8)	7.89(8) 9.1(2)		7.547(3)	0.0012
^{13}C	$(\frac{1}{2}^-,\frac{1}{2}?)$	8	11(1)	10.7(2.9)(0.6)	10(4)	13.3(3) 15.1(5)		8.86(2)	0.15
^{13}C	$(\frac{3}{2}^-,\frac{1}{2})$	–	–	–	–	–		9.897(5)	0.026
^{13}C	$(\frac{7}{2}^-,\frac{1}{2})$	–	–	–	–	–		10.753(4)	0.055
^{13}C	$(\frac{1}{2}^-,\frac{1}{2}?)$	6	–	14(13)(1)	15(13)	14.2(3) 15.3(5)		11.080(5)	< 0.004
^{13}C	$(\frac{3}{2}^-,\frac{1}{2}?)$	6	–	12.0(8.0)(0.6)	7(2)	15.12(8) 16.0(3)		11.75(1)	0.11
^{13}C	$(\frac{5}{2}^-,\frac{1}{2}?)$	6	–	18.3(10.0)(0.5)	19(11)	28.0(7) 30(1)		12.13(5)	0.08
^{13}C	$(\frac{3}{2}^-,\frac{1}{2}?)$	6	–	14(15)(1)	19(10)	23.3(6) 25.4(9)		12.19(1)	0.15
^{13}C	$(\frac{7}{2}^-,\frac{1}{2})$	6	–	14.8(12.4)(0.5)	16(12)	21.7(7) 23.5(8)		12.44(1)	0.14
^{14}B	$(2^-,2)$	8	84(3)	87(4)	83(5)	–		85.43(2)	12.5 ms
^{14}C	$(0^+,1)$	8	112(1)	114(6)	112(3)	–		105.284	5700 years
^{14}C	$(0^+,1)$	8	–	–	–	–		6.589	66 ps
^{14}C	$(2^+,1)$	8	9(1)	9(7)(1)	8(6)	12.2(4) 14.0(5)		7.012	9.0 fs
^{14}C	$(2^+,1)$	8	–	–	–	–		8.318	0.003
^{14}C	$(4^+,1)$	8	–	–	–	–		10.736	0.02
^{14}C	$(1^+,1)$	8	13(1)	12(7)(1)	12(6)	16.7(5) 18.8(5)		11.306	0.046

Table 1. (Continued)

^{14}N	$(1^+,0)$	8	114(2)	116(5)	114.8(7)	–		104.659	stable
^{14}N	$(1^+,0)$	8	7(2)	6.7(5.3)(0.9)	7(5)	10.4(5) 12.3(6)		3.948	4.8 fs
^{14}N	$(1^+,0)$	8	–	–	–	–		6.2035	111 fs
^{14}N	$(3^+,0)$	8	–	–	–	–		6.4462	430 fs
^{14}N	$(2^+,0)$	8	8(2)	8(8)(1)	9(4)	10.39(8) 12.0(3)		7.029	3.7 fs
^{14}N	$(5^+,0)$	8	–	–	–	–		8.964	73 fs
^{14}N	$(2^+,0)$	8	–	–	–	–		8.980	0.008
^{15}N	$(\frac{1}{2}^-,\frac{1}{2})$	8	128(1)	131(8)	129(2)	–		115.494	stable
^{16}O	$(0^+,0)$	8	144(1)	145(8)	145(3)	–		127.619	stable

A similar situation is observed in ^{12}B. Its ground state is known from experiment to be a 1^+ state, the first excited state is a 2^+ state with an excitation energy of about 1 MeV. In our calculations these states extrapolate to nearly the same energy with uncertainties exceeding the energy difference between the levels.

It is conventionally supposed [73, 74, 19] that one cannot describe the spin of the ^{10}B ground state without the use of NNN forces. The ^{10}B ground state is known to be a 3^+ state and there are two low-lying 1^+ states with excitation energies about 700 keV and 2.15 MeV in the spectrum. These 1^+ states are mixed in the NCSM calculations and they cross each other as $\hbar\Omega$ increases. One should be very careful in separating these states and assigning them to two different sets of states that should be extrapolated independently. A small change in this state assignment can result in large changes in their extrapolated energies. In our calculations with a purely two-nucleon interaction JISP16, we obtain the correct spin of the ground state and one of the 1^+ states lies just above it. However, due to the extrapolation uncertainties, we cannot make an unambiguous statement that JISP16 generates the 1^+ state above the 3^+; moreover, the change in state assignment can change the ordering of these states. It is interesting to note that in the GFMC calculations with AV18 NN and IL7 NNN forces, the uncertainties of theoretical results also prevent unambiguous confirmation of the correct ordering of these states (see Table I).

Generally, the JISP16 NN interaction provides a good description of binding energies of nuclei with $N \approx Z$ up to $A = 12$ and overbinds nuclei at the end of p shell. This is illustrated by Fig. 4 where we present some of the results from Table I. It is seen that s-shell nuclei and those at the beginning of the p shell are slightly underbound, the difference with the experimental binding does not exceed a few percent. Starting from $A = 12$, nuclei with $Z \approx N$ are overbound and the difference with experiment increases with A. In particular, ^{16}O is overbound by more than 13 %. Another drawback of JISP16 interaction is that it predicts a faster decrease of binding energies in a set of isobars at a fixed value of A with increase of the isospin I of their ground state.

Rms deviations of the JISP16 binding energies predictions from the experiment are presented in Table 3. The rms deviation of absolute energies E_i^{th} from experimental values E_i^{exp} for nuclei with $A \leq 12$ is about 1.7 MeV for Extrapolation B and about 2.3 MeV for Extrapolation C. These deviations essentially increase by adding all other examined nuclei with $12 < A \leq 16$ to approximately 5.5 MeV. However we study nuclei in a wide range of masses. A 1 MeV discrepancy in description of ^{3}H which has a binding energy of about 8 MeV will be interpreted as a significant failure of the theory while the same 1 MeV discrepancy in description of ^{16}O bound by approximately 130 MeV will be treated

Table 2. Binding energies and spectra of unnatural parity states. The first line for each nucleus gives binding energies for the ground state which is a state of natural parity; the next line gives binding energies for the lowest state of unnatural parity; all the remaining lines for this nucleus show excitation energies of excited states of unnatural parity relative to the lowest state of the same parity. The only exception is ^{11}Be whose ground state given in the first line is of unnatural parity. See Table 1 for other details

	(J^π, T)	$N^M_{\max}$	A	B	C	Av. $\hbar\Omega$ Av. $N_{\max}$	AV18/IL7	Exp.	Width
^{8}Be	$(0^+, 0)$	10 [12]	55.2(5)	55.35(3) [55.5(3)]	54.8(9)	–	56.3(1)	56.500	$6 \cdot 10^{-6}$
^{8}Be	$(2^-, 0?)$	11	36(1)	35.9(3)	34(2)	–		37.59	0.122
^{9}Be	$(\frac{3}{2}^-, \frac{1}{2})$	10 [12]	57.0(6)	57.09(5) [57.2(3)]	56.8(6) [57.0(2)]	–	58.1(2)	58.167	stable
^{9}Be	$(\frac{1}{2}^+, \frac{1}{2}?)$	11	54(1)	53.8(6)	52(1)	–		56.49(2)	0.214
^{9}Be	$(\frac{5}{2}^+, \frac{1}{2}?)$	11	2(1)	1.62(80)(1)	2(1)	1.47(2) 1.32(4)		1.372(9)	0.282
^{10}Be	$(0^+, 1)$	10	64.1(5)	64.0(4)	64.0(4)	–	64.1(3)	64.979	$1.5 \cdot 10^{6}$ years
^{10}Be	$(3^-, 1)$	–	–	–	–	–		57.608(1)	0.016
^{10}Be	$(3^-, 1?)$	9	54(3)	54(2)	52(3)	–		54.83(2)	0.296
^{10}B	$(3^+, 0)$	10	63.8(5)	63.7(3)	63.6(4)	–	64.8(9)	64.753	stable
^{10}B	$(2^-, 0)$	9	56(3)	56(2)	55(3)	–		59.643	0.001
^{10}B	$(3^-, 0?)$	9	1(3)	1.34(2.14)(0.04)	2(3)	1.40(2) 1.35(2)		1.019(3)	0.0015
^{10}B	$(1^-, 0?)$	9	2(3)	1.5(1.5)(0.5)	2(3)	2.82(7) 3.07(6)		1.765(5)	0.120
^{10}B	$(1^-, 1?)$	9	3(3)	3.4(1.7)(1.3)	2(3)	2.8(2) 2.8(1)		2.318(5)	0.094
^{10}B	$(2^-, 0?)$	9	3(3)	3.2(2.0)(0.2)	3(3)	3.87(5) 4.10(5)		2.369(2)	0.074
^{10}B	$(2^-, 1?)$	9	3(3)	3.35(1.40)(0.04)	3(3)	3.2(1) 3.1(1)		2.64(3)	0.21
^{10}B	$(1^-, 0?)$	–	–	–	–	–		2.70(2)	0.26
^{10}B	$(3^-, 1?)$	9	5(3)	4.59(1.58)(0.02)	4(3)	4.32(9) 4.15(7)		3.777(3)	0.096
^{10}B	$(2^-, 1)$	9 [7]	–	4(2)(1)	[8.5(5)]	6.0(2) 6.9(2)		14.18	0.19
^{11}Be	$(\frac{1}{2}^+, \frac{3}{2})$	11 [9]	[62(3)]	63.3(8)	60(3)	–		65.483	13.8 s
^{11}Be	$(\frac{5}{2}^+, \frac{3}{2}?)$	11 [9]	[2(3)]	1.7(1.9)(0.2)	1(2)	1.58(8) 1.43(7)		1.783(4)	0.1
^{11}Be	$(\frac{1}{2}^+, \frac{3}{2}?)$	11 [9]	[8(3)]	8(2)(1)	8(4)	9.3(3) 9.8(2)		5.85(1)	0.139
^{11}B	$(\frac{3}{2}^-, \frac{1}{2})$	10	76.0(4)	76.1(4)	76.1(8)	–		76.206	stable
^{11}B	$(\frac{1}{2}^+, \frac{1}{2}?)$	9	66(3)	66(2)	64(3)	–		69.414	$39 \cdot 10^{-8}$
^{11}B	$(\frac{5}{2}^+, \frac{1}{2}?)$	9	0.4(3.0)	0.7(1.5)(0.2)	1(2)	0.114(9) -0.09(5)	–	0.494	$1 \cdot 10^{-6}$
^{11}B	$(\frac{3}{2}^+, \frac{1}{2}?)$	9	2(3)	1.36(3.09)(0.03)	2(3)	1.93(2) 2.03 (3)	–	1.186	$1 \cdot 10^{-6}$
^{11}B	$(\frac{7}{2}^+, \frac{1}{2}?)$	9	2(3)	1.9(1.3)(0.3)	1(2)	0.82(8) 0.4(1)		2.392(1)	$1 \cdot 10^{-6}$
^{11}B	$(\frac{5}{2}^+, \frac{1}{2}?)$	9	2(3)	1.6(2.0)(0.2)	2(2)	1.38(2) 1.25(3)	–	2.480(1)	0.004
^{11}B	$(\frac{3}{2}^+, \frac{1}{2}?)$	–	–	–	–	–	–	3.081(4)	0.109
^{11}B	$(\frac{7}{2}^+, \frac{1}{2}?)$	9	4(3)	4.0(2.4)(0.1)	4(2)	4.15(4) 4.21(2)		3.810(4)	0.091
^{11}B	$(\frac{9}{2}^+, \frac{1}{2}?)$	9	4(3)	3.9(1.7)(0.3)	4(2)	2.89(9) 2.4(1)		4.47(1)	0.11
^{11}B	$(\frac{5}{2}^+, \frac{1}{2}?)$	–	–	–	–	–	–	4.80(2)	0.18
^{11}B	$(\frac{1}{2}^+, \frac{3}{2}?)$	9	5(2)	5.7(1.6)(0.2)	6(4)	4.9(1) 4.5(2)		5.75(1)	0.205
^{11}B	$(\frac{5}{2}^+, \frac{3}{2})$	–	–	–	–	–		7.54(2)	0.253
^{12}B	$(1^+, 1)$	8 [10]	79(2)	80(2) [80.0(5)]	78.80(7) [79(3)]	–		79.577(2)	20.2 ms
^{12}B	$(2^-, 1?)$	9	–	76(2)	73(6)	–		77.903(1)	<35 fs
^{12}B	$(1^-, 1?)$	9	–	1.0(2.1)(0.2)	1(3)	1.54(4) 1.78(7)		0.94	<49 fs
^{12}B	$(3^-, 1?)$	9	–	1.4(2.1)(0.1)	1(2)	1.50(5) 1.43(5)		1.715(2)	$3 \cdot 10^{-6}$
^{12}B	$(1^-, 1?)$	9	–	1.5(1.7)(0.3)	1(4)	3.27(4) 4.4(2)		2.627(7)	0.009
^{12}B	$(4^-, 1?)$	9	–	3.1(2.4)(0.2)	3(2)	3.19(4) 3.08(5)		2.844(8)	0.11
^{12}B	$(3^-, 1?)$	9	–	3.53(2.51)(0.05)	3(2)	4.40(7) 5.1(2)		4.052(8)	0.05
^{12}B	$(2^-, 1?)$	9	–	5.5(2.3)(0.4)	4(6)	7.1(1) 8.0(2)		5.99	0.045
^{12}B	$(1^-, 1?)$	9	–	5.0(2.6)(0.5)	6(4)	7.64(5) 9.2(4)		6.16(2)	0.06
^{12}B	$(3^-, 1?)$	9	–	6.6(2.4)(0.2)	7(4)	8.35(7) 9.3(2)		6.566(3)	0.065
^{12}B	$(1^-, 1?)$	9	–	5.9(4.4)(0.4)	8(2)	7.51(3) 7.84(6)		7.36(2)	0.095
^{12}B	$(3^-, 1?)$	9	–	7.0(2.3)(0.4)	7(4)	8.1(9) 8.6(6)		7.911(5)	0.034
^{12}C	$(0^+, 0)$	10	94.9(4)	94.8(4)	95.3(3)	–	93.3(4)	92.163	stable
^{12}C	$(3^-, 0)$	9	79(3)	80(2)	79(4)	–		82.522(5)	0.034
^{12}C	$(2^-, 0)$	9	3(2)	2.5(2.4)(0.1)	2(2)	2.8(1) 2.8(1)		2.19(2)	0.26
^{12}C	$(2^-, 1)$	9	7(2)	6.7(1.8)(0.1)	6(3)	5.88(4) 5.48(7)		6.93	0.3
^{12}C	$(3^-, 1)$	9	8(2)	8.1(2.2)(0.1)	8(3)	7.35(6) 6.86(9)		8.71(5)	0.22
^{12}C	$(1^-, 1)$	9	8(2)	7.7(2.2)(0.2)	8(4)	7.44(8) 7.28(8)		12.76(4)	0.275
^{12}C	$(1^-, 1)$	–	–	–	–	–		13.88(3)	0.23

Table 3. Rms deviations of theoretical binding energies from experimental data. For comparison with GFMC AV18/IL7 results, we select only nuclei for which the AV18/IL7 results are available (see Tables 1 and 2)

	Number of nuclei	JISP16		AV18/IL7
		Extrap. B	Extrap. C	
Comparison of JISP16 and AV18/IL7 results				
Absolute energies (MeV)	13	1.16	1.44	0.43
Relative energies	13	0.023	0.029	0.007
Energies per nucleon (MeV)	13	0.12	0.16	0.04
$A \leq 12$ ground states				
Absolute energies (MeV)	19	1.68	2.33	–
Relative energies	19	0.033	0.047	–
Energies per nucleon (MeV)	19	0.16	0.22	–
All ground states				
Absolute energies (MeV)	26	5.63	5.49	–
Relative energies	26	0.055	0.059	–
Energies per nucleon (MeV)	26	0.39	0.39	–

as a significant success of the theory. Therefore we present in Table 3 also the rms deviations of the energy per nucleon predictions, i. e., rms deviations of E_i^{th}/A from the respective experimental data. In addition, we also present in Table 3 the rms for relative energies $(E_i^{th} - E_i^{exp})/E_i^{exp}$. We believe that these rms deviations are more instructive for the statistical analysis of energies when nuclei differ widely in mass.

Table 3 also compares the JISP16 binding energy predictions with those of GFMC obtained with AV18 NN and IL7 NNN interactions. For this comparison, we use only 13 nuclei for which the AV18/IL7 results are available (see Table 1). The AV18 + IL7 strong interaction model appears to be more accurate in description of light nuclei bindings. However the JISP16 predictions seem reasonable: the rms deviations of relative energies for all 26 nuclei studied does not exceed 6 %.

5.2. Spectra of Excited States

The energies of excited states can be obtained by the same Extrapolations A, B and C as of the ground states. These results are summarized in Tables 1 and 2. Note that, for excited states, we present in the Tables excitation energies which are, of course, the difference between the energy of the given state and the ground state energy. For natural parity states in ^{11}Be where the ground state has unnatural parity and for unnatural parity states in other nuclei, we present excitation energies relative to the lowest state of the same parity. The numbers in brackets after excitation energies shown in the Tables are evaluated as uncertainties of the respective absolute energy extrapolation of the excited state.

Our statistical analysis is presented in Table 4. We see that JISP16 provides a less accurate description than the AV18 + IL7 interaction model of 38 states in light nuclei for which AV18/Il7 results are available. However, even for those states, the rms deviation of relative

Table 4. Rms deviations of theoretical level energies from experimental data. For comparison with GFMC AV18/IL7 results, we select only states for which the AV18/IL7 results are available (see Tables 1 and 2)

	Number of levels	JISP16		AV18/IL7
		Extrap. B	Extrap. C	
Comparison of JISP16 and AV18/IL7 results				
Absolute energies (MeV)	38	1.38	1.8	0.55
Relative energies	38	0.03	0.04	0.009
Energies per nucleon (MeV)	38	0.16	0.21	0.05
Natural parity states				
Absolute energies (MeV)	96	3.71	3.97	–
Relative energies	96	0.043	0.05	–
Energies per nucleon (MeV)	96	0.28	0.31	–
Natural and unnatural parity states				
Absolute energies (MeV)	135	3.54	4.04	–
Relative energies	135	0.05	0.056	–
Energies per nucleon (MeV)	135	0.28	0.34	–

energies for JISP16 predictions does not exceed 4 %. For all 133 calculated states obtained with JISP16, the rms deviation of relative energies is about 5 % for Extrapolation B and about 6$\dot{\%}$ for Extrapolation C. Hence JISP16 can be regarded as a realistic NN interaction providing a reasonable description of level energies in light nuclei.

The JISP16 predictions for the level ordering in the spectra is correct in most cases. However, evaluated uncertainties of level energies are sometimes close to or even larger than the splitting between levels. We have already discussed this issue while examining predictions for ground state spins of a few nuclei. One can however expect that the extrapolation uncertainties in Tables 1 and 2 overestimate the uncertainties of excitation energies. All states of the same parity are calculated in the same NCSM run. Truncation of the basis space in each run can cause an overall shift of the whole spectrum with a smaller effect in energy difference between levels, i. e., in excitation energies. In this case the extrapolation uncertainties may mostly reflect this overall shift of the spectrum rather than the uncertainties of excitation energies. Unfortunately, we cannot prove this expectation and we are unable to suggest an accurate estimate of excitation energy uncertainties. As a rough estimate of these uncertainties, we suggest in the second set of brackets after Extrapolation B excitation energy results in Tables 1 and 2 the rms deviation from the quoted value of the excitation energy obtained by the same extrapolation with the largest available $N_{\max}$ by varying $\hbar\Omega$ in a reasonable interval that is a 10 MeV interval of $\hbar\Omega$ values which includes the minimum of the NCSM results for the absolute energy of the given state. A small number in these second set of brackets and hence the stability of Extrapolation B with regard to the $\hbar\Omega$ variation confirms our expectation of the overall shift of the spectrum and possible reduction of the excitation energy uncertainty. We see from the Tables that in many cases the number in the second set of brackets is smaller, sometimes by an order of magnitude, than the uncertainty estimate based on absolute energy given in the first set of brackets.

Table 5. Rms deviations of theoretical excitation energies from experiment. For comparison with GFMC AV18/IL7 results, we select 25 states for which the AV18/IL7 results are available (see Table 1). For unnatural parity states, we calculate excitation energy relative to the lowest state of the same parity

	Number of levels	JISP16				AV18/IL7
		Extrap. B	Extrap. C	Av. $\hbar\Omega$	Av. N_{max}	
Natural parity states						
Excitation energies (MeV)	25	0.61	0.74	0.96	1.60	0.42
Excitation energies (MeV)	70	1.62	2.88	3.49	4.43	–
Unnatural parity states						
Excitation energy (MeV)	31	2.25	1.57	2.05	2.32	–

However, there are states for which these numbers do not differ essentially, and in these cases we should not expect a reduction of the uncertainty estimated from the absolute energy extrapolation.

Extrapolations A and B provide, as a rule, consistent predictions for excitation energies which sometimes differ from Extrapolation C predictions. This is illustrated by Fig. 5 where we plot spectra of a few light nuclei. Our impression is that Extrapolations A and B provide more reliable results at least in cases of sufficiently large $N_{\max}$ values. Note, Extrapolations A and B employ information from NCSM results around the minima of their $\hbar\Omega$ dependence which lie close to the extrapolated energy. Hence their predictions may be expected to be more accurate than the predictions of Extrapolation C based on NCSM results which lie higher in energy. Rms deviations of excitation energies of natural parity states from Extrapolation B are essentially smaller than those from Extrapolation C as is seen in Table 5. For excitation energies of unnatural parity states, we calculate excitation energies relative to the lowest state of the same parity. Surprisingly, Extrapolation C results in much smaller rms deviations of excitation energies of unnatural parity states than Extrapolation B (see Table 5). A possible explanation may rely on the fact that unnatural parity states were generally calculated with smaller $N_{\max}$ than natural parity states: there are no narrow unnatural parity states in nuclei with $A < 8$, most of examined unnatural parity states are in nuclei with $A \geq 10$ where we cannot use large $N_{\max}$ values. In this case, the minima of the $\hbar\Omega$ dependence of the NCSM results are further from the extrapolated energy and the Extrapolation B become less reliable. On the other hand, the Extrapolation C is founded on theory and its predictions may be less affected by the decrease of the basis space.

As compared to the AV18/IL7 predictions, the JISP16 results in larger rms deviations of excitation energies (see Table 5) but still is rather accurate in the description of the spectra of light nuclei.

As was already noted, we obtain all states of a given parity in each NCSM run and thus obtain excitation energies of all excited states of this parity directly from the NCSM results. These excitation energies may depend on $\hbar\Omega$ and $N_{\max}$. However, if these dependences are weak, one can get a reasonable estimate of excitation energies (see discussions in Refs. [35, 46]). We present in Tables 1 and 2 excitation energies obtained by averaging for a

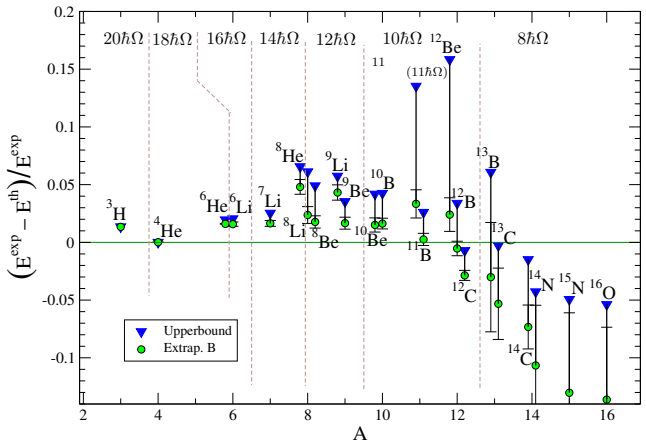

Figure 4. Relative error in description of binding energies (as compared to experiment) of light nuclei. Triangles — variational upperbounds for the ground state energies obtained directly from NCSM calculations; circles — Extrapolation B results with estimated uncertainties.

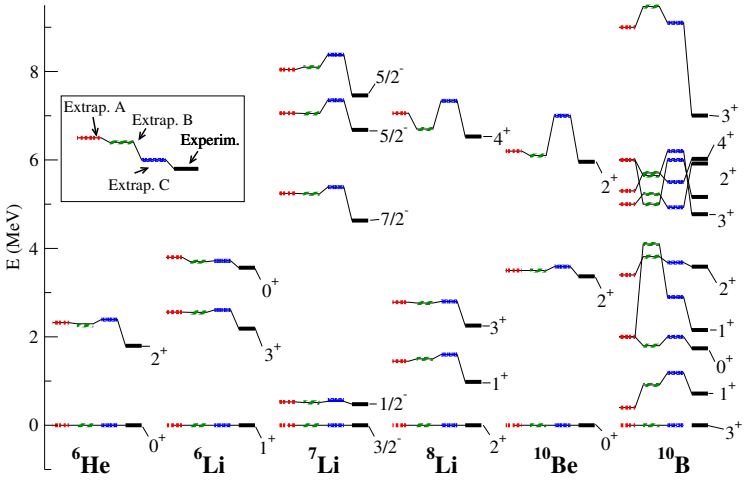

Figure 5. Spectra of a few light nuclei obtained by Extrapolations A, B and C in comparison with experiment. The ground state energies are all shifted to zero energy in order to compare excitation energies.

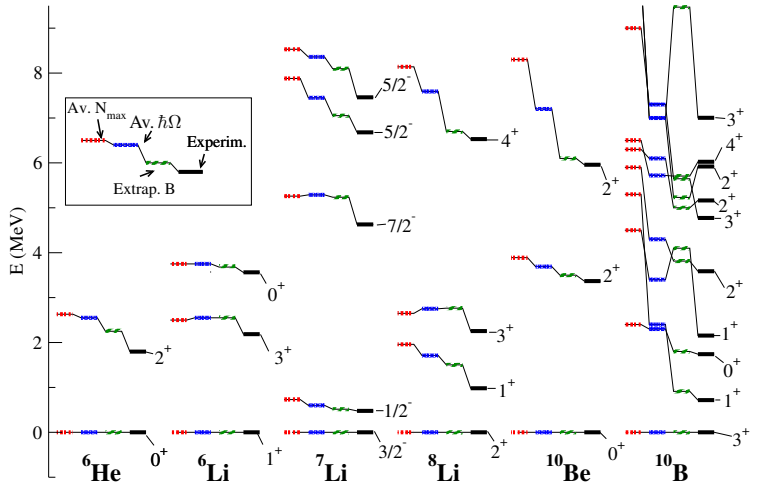

Figure 6. Spectra of a few light nuclei obtained by averaging excitation energies of a given level obtained directly in the NCSM calculations within a reasonable interval of $\hbar\Omega$ values with the largest available $N_{\max}$ value, $N_{\max} = N^M_{\max}$ (Av. $\hbar\Omega$), and with all $N_{\max} = N^M_{\max}$, $N_{\max} = N^M_{\max} - 2$, $N_{\max} = N^M_{\max} - 4$, $N_{\max} = N^M_{\max} - 6$ (Av. $N_{\max}$) and by Extrapolation B in comparison with experiment.

given level the NCSM excitations from calculations with largest available $N_{\max} = N^M_{\max}$ in a reasonable interval of $\hbar\Omega$ values (usually a 10 MeV interval around minima of $\hbar\Omega$ dependencies for the given and lowest state of the same parity) in the 'Av. $\hbar\Omega$' column with a rms deviation of these excitation energies from the average in round brackets. As it was shown in Ref. [75] (see Ref. [63] for a more detailed discussion), wide resonances should have correspondingly strong $\hbar\Omega$ dependence of excitation energies that should result in larger values of these rms deviations. Therefore the number in brackets is useful for selecting narrow resonances. In the 'Av. $N_{\max}$' column of Tables 1 and 2, we present the result of averaging excitation energies of a given level obtained directly in the NCSM calculations with $N_{\max} = N^M_{\max}$, $N_{\max} = N^M_{\max} - 2$, $N_{\max} = N^M_{\max} - 4$ and $N_{\max} = N^M_{\max} - 6$ within a reasonable interval of $\hbar\Omega$ values selected in the same manner as in Extrapolation A with a rms deviation of these excitation energies from the average in round brackets. A large number in the round brackets indicates a strong $N_{\max}$ dependence which means that the respective estimate of the excitation energy is inaccurate. Even more instructive is a difference between values from the columns 'Av. $\hbar\Omega$' and 'Av. $N_{\max}$' — this estimate of the excitation energy is reliable only if that difference is small.

We illustrate this method of calculation of excitation energies in Fig. 6 where we compare the spectra of a few light nuclei obtained by it and by Extrapolation B. It is seen that levels for which the excitation energies from 'Av. $\hbar\Omega$' and 'Av. $N_{\max}$' columns are close are described consistently with Extrapolation B. Otherwise we have a large difference with predictions based on Extrapolation B. Generally, the results of averaging excitation energies obtained with maximal attainable $N_{\max}$ only are closer to the predictions based on extrapolations than the results of averaging states obtained with a set of $N_{\max}$ values.

In the case of natural parity states, the numbers in the columns 'Av. $\hbar\Omega$' and 'Av. $N_{\max}$' are significantly different for many states. As a result, the statistical analysis of all excited states based on these numbers without special selection of states brings us to large rms deviations with experiment shown in Table 5. Remarkably, in the case of unnatural parity, the numbers in the columns 'Av. $\hbar\Omega$' and 'Av. $N_{\max}$' of Table 2 are consistent nearly for all states. Therefore averaging NCSM excitation energies is more suitable for studying spectra of unnatural parity states. Statistical analysis shows that rms deviations with experiment for spectra of unnatural states based on numbers from columns 'Av. $\hbar\Omega$' and 'Av. $N_{\max}$' are even smaller than rms deviations for Extrapolation B.

6. Conclusion

We present here *ab initio* results for binding energies and spectra of 26 s- and p-shell nuclei with the JISP16 NN interaction. The calculations are performed using the NCSM with further extrapolation of energies to the infinite basis space, along with uncertainty quantification, in accord with NCFC. Our results are compared wherever possible with GFMC results obtained with AV18 NN and IL7 NNN forces.

Generally, JISP16 provides a reasonable description of s- and p-shell nuclei. Binding energies are well described with the exception of few nuclei at the end of p shell where JISP16 overbinds nuclei by approximately 10 %. The spin of ground states is correctly reproduced with a few possible exceptions where the excitation energy of the lowest excited state is less than the uncertainty of theoretical predictions. The spectra of narrow states are also reasonably well reproduced. However, we see that the AV18 + IL7 strong interaction model provides a somewhat better description of energies in light nuclei. On the other hand, JISP16 does not require three-nucleon forces that makes it possible to use larger basis spaces in calculations thus reducing uncertainties of the theory and making it possible to explore heavier nuclear systems.

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