Review of the book

"Dynamic Shapley Value and Dynamic Nash Bargaining"

by David Yeung and Leon Petrosyan

The book "Dynamic Shapley Value and Dynamic Nash Bargaining" of Professor Yeung and Professor Petrosyan provides a new evidence of their indisputable scientific authority in the field of Dynamic Games theory and applications. The content of the book is brilliantly highlighted by Professor Mazalov in his brief but very informative foreword. He explains that the contribution of the book is the dynamic extension of two milestone results in Cooperative Games, the bargaining solution of Nash (in cooperative games with nontransferable payoffs) and the value of Shapley (in cooperative games with transferable payoffs). Real-life processes in economics, biology, social life, etc. are inherently dynamic, comprise multiple agents with their own aspirations and, nevertheless, inevitable need to cooperate with each other. Therefore I believe that mathematical results presented in the book will have a long echo in various applications in different and unexpected fields.

For my part, I would like to emphasize a stylistic value and a pedagogical significance of the book. It is a book which is very well thought out and very well written. All the content is structured in a reader-friendly manner making easier to follow and to perceive complicated mathematical matters. These features make the book a perfect textbook for the students and for any interested reader.

In this book, not a word is said for nothing. In Chapter 2, we get reminded (or familiar) with three fundamental theorems of dynamic optimization presented in the discrete-time setting: deterministic dynamic programming of Bellman, its stochastic version and its version with a random horizon. In the next chapters, these ideas are adapted and interpreted in the dynamic games framework. It looks like a spectacular demonstration of how old ideas return in new content. For example, the dynamic programming technics

(Theorem 1.1 of Chapter 2) applies in the proof of Theorem 1.1 of Chapter 3 (the feedback Nash dynamic game equilibrium), Theorem 2.1 of Chapter 4 (the equilibrium solution of the coalition game based on Shapley value), and Theorem 2.2 of Chapter 5 (optimal cooperative strategies under constant weights). Similarly, the other basic facts of Chapter 2 are propagated through the other parts of the book.

Chapter 3 presents the general structure of a cooperative game with transferable payoffs, as well as fundamental principles of its solution: group optimality, an individual rationality and a subgame consistency. These principles justify the logics of including the results of Chapter 2 into the dynamic game framework. The group optimality principle yields a non-cooperative Nash equilibrium solution. The individual rationality principle requires such a scheme of payoff distribution between the participants that their individual payoff under cooperation should not be less than that without cooperation. This can be realized by different optimality principles yielding different subgame consistent payment mechanisms. One of such mechanisms is established in Theorem 2.1 of Chapter 3. The stochastic version of these results is also presented based on the stochastic dynamic programming technics.

In Chapter 4, the specific optimality principle, namely the Shapley value imputation, is applied in order to derive the corresponding payoff distribution procedure. This constitutes the first of two main contributions of the book, the dynamic Shapley value. The main result of the chapter is formulated in Theorem 2.2 providing an explicit formula for dynamic payoff distribution. Chapter 5, devoted to dynamic games with nontransferable payoffs, contains the second main contribution of the book, namely, the results on dynamic Nash bargaining. By invoking again the dynamic programming approach, the dynamic Nash bargaining with variable weights is derived and summarized by Theorem 3.1. Stochastic extensions of dynamic Shapley value imputation and dynamic Nash bargaining are highlighted in Chapter 6.

As it should be in a good textbook, in the book of Yeung and Petrosyan, all theoretical statements are illustrated by meaningful and interesting

examples. The examples (the dynamic resource optimization problem, the resource game, the investment game, and others) are elaborated in all details providing explicit problem solutions. The same examples are consequently enriched with new features (for example, a deterministic setting is changed to a stochastic) thus developing at the reader a unified look at the problems. It should be specially mentioned the example of the investment game in Chapter 4, illustrating the dynamic Shapley value (the detailed derivations on 19 pages), and the example of the public capital build-up problem, illustrating the dynamic Nash bargaining (15 pages of the book). Such detailed presentation of the examples will be gratefully accepted by the reader. The same can be said on the problems in the end of the chapters which are intended for an independent solution.

In conclusion, I am happy to recommend the book of David Yeung and Leon Petrosyan to the students and to the specialists for a thoughtful slow reading. In the end, you will see that it was worth it. You will get new knowledge on such an interesting topic as Dynamic Cooperative Games theory and new inspirations for the future.

Prof. Vladimir Turetsky
Ort Braude College of Engineering
Karmiel, Israel