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Chapter 2

OLD MATH AND RENEWED PHYSICS: KEYS TO UNDERSTANDING GRAVITY

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Abstract

This chapter develops a new approach to the central subject matter of GRT: the phenomenon of gravity. It pursues a concept of gravity as a statistical residue from electromagnetic interactions between members in a population of overall charge-neutral atoms. Such interactions are similar to interactions between neutral current elements: some are attractive, and some are repulsive. More than century ago, J.M. Ampère studied forces between macroscopic current elements, and characterized them mathematically in great detail. His theory has never been falsified experimentally, although it has been eclipsed theoretically, by a different theory, about different subject matter: Maxwell's theory of electromagnetic fields and their actions on charged particles. In modern times, Ampère's experiments have been successfully repeated and extended. So even though his theory has been largely sidelined, it remains viable in all experiments, and awaits a revival. So it is here enlisted for the study of the gravity problem. The statistical aspects of the study come from the modern Statistical Mechanics of classical gasses, quantum photons, and elementary material particles. There are also computational aspects to the study, and they too reach into history: Plato's ideal regular polyhedra help solve a big statistical sampling problem.

Introduction

General Relativity Theory (GRT) is our current best approach to the subject of gravity, but the present realization of GRT cannot serve as the only approach that we will ever use. The problems left by present-day GRT include: 1) The observational tests available are few in number, and great in technical difficulty; 2) The foundation on which present-day GRT rests is Special Relativity Theory (SRT), and the foundation on which SRT itself rests is under continuing study; 3) A long-sought unification of GRT, or even SRT, with Quantum Mechanics (QM) is not yet accomplished, and does not look imminent.

So what ideas might help us develop some additional techniques for study of the subject of gravity? I believe that a short review of ways in which science overall has developed over the centuries can provide some guidance for the near-term development of such additional techniques.

Science has always had two important objectives that stand somewhat in opposition to each other. One is about ‘Divide and Conquer’, and it applies to the set of phenomena that Science addresses. The other is about ‘Unify and Simplify’, and it applies to the suite of theories that Science applies. Often the pursuers of either objective create more work for the pursuers of the other objective, and that result can be very welcome, or somewhat unwelcome, depending on circumstances. But in the long run, the exchange brings progress.

The Divide and Conquer objective tends to arise in response to new observations, or new experiments, and so it tends to lead to proliferation of new theories. The Unify and Simplify objective tends to arise in response to the perception of having to many, and possibly redundant, or even conflicting, theories, and so it may challenge older theories, and can eventually reduce the number of theories still routinely used.

The present chapter honors both traditions. It revives an older theory that is not currently in wide use, and it addresses the desire among physicists to see more unification in present-day Physics.

The older theory here revived is that of J.M. Ampère concerning interactions between charge-neutral current elements. The theory amounted to a mathematical description of observed phenomena, including movement, deformation and shattering of metals – even without any evidence of melting, or even heating.

Ampère worked in early to mid-19th century France. His work has since been largely eclipsed with that of J.C. Maxwell, who worked later in 19th century England. Maxwell concentrated on the electromagnetic fields arising from charged particles, at rest and in motion. It is possible to apply Maxwell’s equations in Ampère scenarios, but it is awkward, because one must cycle back and forth between the field equations, and the Lorentz force law for the effect of the fields on mobile charges.

In the 20th century and beyond, the British scientist Peter Graneau, working both there and in America, revisited Ampère’s experiments and wrote extensively about them and their implications; see, for example, his book written with his son Neal, [1]. The phenomena discussed involve currents, and their effects on the wires carrying them. Extreme currents put wires into tension, and can deform them, or even shatter them. I believe it happens because mobile electrons are driven toward the surface of the wire, leaving the interior positively charged, and self-repellant.

Except for Graneau and his colleagues, most modern researchers have not considered using Ampère’s old theory on modern problems. This is the case largely because Ampère’s theory is so Newtonian in character: the forces are modeled as if they were instantaneous. This aspect of Ampère’s theory seems to make it incompatible with Einstein’s Special Relativity Theory (SRT); see [2]. By contrast, Maxwell’s theory about fields and charges is fully compatible with SRT - and indeed with *any* theory that can be written in tensor notation.

That incompatibility between Ampère’s theory and SRT should not, however, be considered a fatal defect. Consider the title of the present paper: it includes the words ‘Old Math & Renewed Physics’, which appeared also in the title of an earlier Chapter in an earlier Nova book [3]. The title of that book has the word ‘Challenges’ in it. One of our present-day Challenges arises from SRT: Einstein’s derivation of SRT did not use the typical 19th century mathematical approach, consisting of: 1) differential equations; 2) families of solutions; 3) application of boundary conditions. His alternative approach violated some advice dating back to Euclid; i.e., do *not* be injecting new Postulates into a situation where an established and reliable approach already exists.

What then really *does* matter? We normally think of gravity as an interaction between charge-neutral macroscopic bodies, such as planets and stars. So Ampère’s focus on charge-neutral matter is certainly appropriate for gravity. In addition, on a micro scale, all those macroscopic bodies are made of atoms, and any atom possesses internal moving charges. That fact makes any atom a lot like an Ampère current element. So Ampère’s focus on forces between current elements is also appropriate for gravity between atoms. Those are the reasons why the previously semi-retired Ampère theory is here resurrected for the study of gravity.

But a *lot* of atoms taken together must all interact with each other, making an *enormously* complicated scenario. Here, the instantaneous character of Ampère’s theory becomes a real asset: it forces us to take the emphasis off of *individual* interactions occurring *in sequence*, and redirect it to a *population* of enormously *many* interactions occurring *in parallel*.

So the present Chapter also invokes a suite of other physical theories that are not in any way retired, but are also not currently recognized as being relevant for gravity research. They comprise the several theories we have about Statistical Mechanics for several kinds of particles. See [4]. With these tools, we can think about gravity less in terms of specifics, and more in terms of statistics. ‘Emergent behavior’ is how we might then characterize the gravity phenomenon. Note that taking this statistical view of gravity puts its study more into harmony with QM than it has previously been. That seems like a step in the right direction: a step toward unification in Physics. The Chapter finally describes an approach for understanding the extreme smallness of the numerical value of the Gravitational constant G. The implication is that gravity is not a separate kind of physics, needing a separate theory distinct from all others; it can be understood in terms of other theories already available.

Relevant Existing Theories

Let us begin with a short catalog of the existing theories possibly relevant to the gravity problem, along with remarks about their particular attributes:

- **Newton’s Mechanics:** The original subject matter was neutral bodies interacting via forces, but later the same math was also applied to charged bodies interacting with electric and magnetic fields. In the Newton scheme of things, the signal speed for all interactions was infinite.
- **Ampère’s Theory:** The subject matter consisted of observed forces acting between charge-neutral current elements in electrical circuits. As in Newton’s mechanics, the signal speed in Ampère’s Theory was presumed infinite.
- **Maxwell’s Electromagnetic Theory (EMT):** The subject matter was electromagnetic fields and charged particles – *not* neutral matter. The four first-order coupled field equations described the interactions between four electromagnetic fields, and the two second-order un-coupled wave equations derived from them defined a finite signal speed *c*.

- **Lorentz's Force Law:** The subject matter was the effect of electromagnetic fields acting on charged particles, both stationary and moving. The effect of electromagnetic fields acting on neutral current elements was *not* explicitly covered. And, of course, the effect of neutral current elements directly on each other was not covered.
- **Einstein's Special Relativity Theory (SRT):** The subject matter was largely Lorentz transformations among differently moving coordinate frames, each with its own particular observer in residence. There was a finite signal *speed*, but there was *not* a finite signal *energy*; the signal was presumed to act the same way that a plane wave of infinite extent, and hence infinite energy, would act.
- **Einstein's General Relativity Theory (GRT):** The subject matter was largely the stress-energy-momentum tensor affecting the metric tensor of unified 'spacetime' in a way that could be enlisted to represent the effect of gravity.
- **Quantum Mechanics (QM):** The subject matter was the obvious quantization observed everywhere in Nature. QM had many fathers, including Einstein, Bohr, Schrödinger, Heisenberg, Dirac, and others, producing many different descriptions for many different problems. Over all of them though, the focus was on quantum *states*, and not on quantum *processes*. So signals were not generally relevant. But one character in the QM drama, namely the *photon*, really had the properties appropriate for a signal: finite speed, and finite energy.
- **Statistical Mechanics (SM):** The subject matter was the macroscopic behavior of systems with staggeringly large numbers of particles. Like QM, SM too had many fathers, including Maxwell & Boltzmann for classical molecular gasses, Fermi & Dirac for electrons and other fractional-spin particles, and Bose & Einstein for photons and other integer-spin particles. QM had changed the subject, away from specific trajectories in time, and toward statistics of state occupancy. The expected population of any state would depend on its so-called Boltzmann factor, which favored lower-energy states. The SM description of systems enlisted the thermodynamic notion of Maximum Entropy.
- **Information Theory (IT):** This theory emerged in the mid twentieth century, pioneered by Claude Shannon, and applied widely in the communication and computation industries. Mathematically, Information is just negative Entropy. So IT recalls Thermodynamics, Entropy, and SM. It makes clear the reason for modeling signals in terms of finite-energy pulses: an infinite plane wave cannot function as a signal because it has no mark of *beforevs*. *after*, and that deficit makes it incapable of conveying any message whatsoever!

The Present Approach

The original Ampère theory dealt with a lot of geometric details that affect the vector force between two current elements. The two current elements lie separated at some distance measured along a line connecting them. The current elements may lie at different angles from the connecting line, and there may be another non-zero angle between the two planes that the two current elements make with respect to the connecting line.

But, is all this detail necessary for the gravity problem? Really, we are not so interested in the vector force that the geometry finally specifies; we are more interested in the scalar potential energy from which the vector force can arise.

Since we generally have a huge collection of neutral atoms, we can think in terms of statistics. The geometric states of neutral particle pairs translate into energy states. We need to formulate statistical ideas about how different possible energy states of neutral particle pairs are populated.

Let variable E represent state energy. Given variable E, the population of particle pairs certainly possesses an average energy E_{avg}.In addition, any individual particle pair has its individual energy E.

One novel thing about gravity modeled in terms of Ampère's theory is that Ampère's theory gives both attractive *and* repulsive forces. So it implies both negative *and* positive energies. This symmetry is appropriate for a gravity model, inasmuch as trajectories in a gravitational field can include both bound *and* unbound situations.

So in the present model, a particle pair has a maximum energy $E_{\max}$, positive, *and* a minimum energy $E_{\min}$, negative. And they are of equal magnitudes, $E_{\min} = -E_{\max}$.But except for having *both* negative and positive energies explicitly involved, there is a lot in the situation that is similar to situations known from traditional Statistical Mechanics.

One feature that emerged early in the development of Statistical Mechanics was the so-called 'Boltzmann factor'. This concept arose in the description of a population of classical gas molecules. They scatter off each other, but the details of their interactions and travel directions are not central to their statistical description. What matters is energy; in particular, kinetic energy. The Boltzmann factor says high kinetic energy is exponentially less likely than low kinetic energy, and even more so for lower temperatures. This feature leads to the Maxwell-Boltzmann distribution for energies of classical ideal gas molecules.

The importance of the Boltzmann factor was re-iterated in the case of the spectrum of blackbody radiation; i.e., thermal radiation. Without the Boltzmann factor, we had the so-called 'Ultraviolet Catastrophe': light intensity expected based on wave vectors possible, and becoming increasingly great for increasingly large wave vectors; i.e., high frequencies. That doesn't actually happen. And, with the Boltzmann factor, we have the perfectly reasonable resolution: the Planck model for the actual blackbody spectrum of thermal radiation.

The Boltzmann factor carries over from the description of classical particles and light to the description of all quantum particles. It appears in the Bose-Einstein statistics for a population light quanta; i.e., photons, and it carries over to other similar particles, generically called 'Bosons'. Bosons are infinitely tolerant of each other, and allow arbitrary numbers in the exact same energy state.

The Boltzmann factor is equally important in the description of the other kind of quantum particle; i.e., electrons and other similar particles, generically called 'Fermions'. Such particles are *not* tolerant, and allow no more than two particles in the same energy state. With that stipulation, we have Fermi-Dirac statistics for electrons and all similar particles. The two particles allowed in one energy state are distinguished by their so-called 'spin', to which values of $\pm 1/2$ are attributed.

In all problems, the Boltzmann factor has the exponential form $\exp(-E/E_{\text{exd}})$, where E_{exd} is the rate of exponential decline. The Boltzmann factor extends over all allowed values of energy E .In general, the range can be arbitrary; i.e., E_{min} to E_{max} .

The original Maxwell-Boltzmann problem was a continuous, whereas modern quantum problems are discrete. The gravitational problem appears to be a continuous one, but there is a long-standing desire to unify general relativity theory with quantum mechanics, and that would suggest some form of quantum gravity, and, in turn, a discrete description. This issue cannot be decided at the present time. So in the following, we consider both possibilities.

For computing statistics, every Boltzmann factor has to be normalized. If E is a continuous variable, the normalization of the probability density function is the integral $\int_{E_{\text{min}}}^{E_{\text{max}}} \exp(-E/E_{\text{exd}})\,dE$ .If E is a discrete variable, the probability normalization is the sum $\sum_{E_{\text{min}}}^{E_{\text{max}}} \exp(-E/E_{\text{exd}})$.

The Average Value of Energy

For a continuous problem, the probability density at energy E is:

$$\exp(-E/E_{\text{exd}}) \Big/ \int_{E_{\text{min}}}^{E_{\text{max}}} \exp(-E/E_{\text{exd}})\,dE \,.$$

For a discrete problem, the probability at energy E is:

$$\exp(-E/E_{\text{exd}}) \Big/ \sum_{E_{\text{min}}}^{E_{\text{max}}} \exp(-E/E_{\text{exd}}) \,.$$

For a typical continuous and classical problem, like the Maxwell-Boltzmann description for a population of gas molecules, the energy is kinetic, which is positive, and the integration limits are $E_{\text{min}} = 0$ and $E_{\text{max}} \to \infty$. In such cases, the average energy E_{avg} satisfies the relation:

$$E_{\text{avg}} = \int_0^\infty E\exp(-E/E_{\text{exd}})\,dE \Big/ \int_0^\infty \exp(-E/E_{\text{exd}})\,dE = E_{\text{exd}} \,.$$

That is, in the Maxwell-Boltzmann case, $E_{\text{avg}} = E_{\text{exd}}$.

But in general, an exponential distribution does not mandate $E_{\text{avg}} = E_{\text{exd}}$. For example, in a typical discrete quantum atom problem, the energies are the sums of kinetic and potential energies for different discrete states of an atom. For a bound system, these state energies are all *negative*. Let Index $n = 1$ correspond to the lowest-energy state. This state is certainly favored. It, and the higher-energy states, are all represented in proportion to their respective Boltzmann factors. The average energy for an atom in the population is then:

$$E_{\text{avg}} = \sum_{n=1}^{n\to\infty} E_n \exp(-E_n / E_{\text{exd}}) \Big/ \sum_{n=1}^{n\to\infty} \exp(-E_n / E_{\text{exd}}) \ .$$

This E_{avg} is certainly negative, and so cannot equal the exponential decline parameter E_{exd}, which is positive. Typically, the parameter E_{exd} reflects the temperature T; one expects $E_{\text{exd}} = K_{\text{B}}T$ where K_{B} is Boltzmann's constant.

For another example, consider the typical discrete photon/ Boson problem. The energies are positive and the summations start with a minimum energy at $n = 0$. There we have the 'zero-point' energy, $E_{\min} = E_{n=0} = \frac{1}{2}\Delta E$, where ΔE is the characteristic size of a quantum. Then $E_n = E_{\min} + n\Delta E$ with integer n going from 0 to ∞. The average energy is:

$$E_{\text{avg}} = \sum_{n=0}^{n\to\infty} E_n \exp(-E_n / E_{\text{exd}}) \Big/ \sum_{n=0}^{n\to\infty} \exp(-E_n / E_{\text{exd}}) \ .$$

This energy is certainly positive. But because of the zero-point involvement, E_{avg} is at least $\frac{1}{2}\Delta E$, but E_{exd} can in principle be any positive value, so we do not have $E_{\text{avg}} = E_{\text{exd}}$.

Now, for the gravitational application, $E_{\min} = -E_{\max}$, $E_{\max}$ is positive but finite, and the value of E_{avg} is different from E_{exd}. One can anticipate that $E_{\text{avg}} < 0$, and that small E_{exd} would make for fast decline, and hence $E_{\text{avg}} \to -E_{\max}$, while large E_{exd} would make for slow decline, and hence $E_{\text{avg}} \to 0$.

Traditionally, the gravity problem has been regarded as a continuous problem, which implies integrals rather than sums. So for the gravity problem, the average energy would satisfy the relationship:

$$E_{\text{avg}} = \int_{-E_{\max}}^{E_{\max}} E \exp(-E / E_{\text{exd}})\, dE \Big/ \int_{-E_{\max}}^{E_{\max}} \exp(-E / E_{\text{exd}})\, dE \ .$$

From this relationship, we can work out what E_{avg} is in terms of $E_{\max}$ and E_{exd}.

First, we can refer to standard tables of integrals to help evaluate both

$$\int_{-E_{\max}}^{E_{\max}} \exp(-E / E_{\text{exd}})\, dE \text{ and } \int_{-E_{\max}}^{E_{\max}} E \exp(-E / E_{\text{exd}})\, dE \ .$$

We have:

$$\int_0^\infty \exp(-x)\,dx = -\exp(-x)\Big|_0^\infty = -0-(-1) = +1 \ ,$$

and

$$\int_0^\infty x\exp(-x)\,dx = -\exp(-x)\Big|_0^\infty = \left[-0-(-1)\right] = +1 \ .$$

In summary,

$$\int_0^\infty \exp(-x)\,dx = 1 \text{ and } \int_0^\infty x\exp(-x)\,dx = 1 \ .$$

So over the infinite interval $x=0$ to $x \to \infty$, the average value of x is:

$$x_{\text{avg}} = \int_0^\infty x\exp(-x)\,dx \Big/ \int_0^\infty \exp(-x)\,dx = 1/1 = 1 \ .$$

That is, the average value of x is unity.

Observe that these standard integrals correspond to $E_{\text{min}} = 0$, $E_{\text{max}} \to \infty$, and $E_{\text{exd}} = 1$. These particulars are convenient for the problems already familiar from Statistical Mechanics, such as atoms in a perfect gas, photons in thermal radiation, and free electrons in metals. All of those problems are single-sided, so $E_{\text{min}} = 0$, and they are unlimited, so $E_{\text{max}} \to \infty$, and E_{exd} is exogenously determined by temperature T and Boltzmann's constant K_{B} : $E_{\text{exd}} = 1$ in units of $K_{\text{B}}T$.

The gravity problem is different from traditional problems in three ways. **1)** Energy is limited to a finite magnitude E_{max} ; **2)** the energy distribution is double-sided, with $E_{\text{min}} = -E_{\text{max}}$. **3)** The exponential decline parameter E_{exd} is not *a priori* known to be equal to $K_{\text{B}}T$ .We now have to develop an approach appropriate for these particulars.

Accommodating a finite upper limit:

With a finite upper limit x_{max} the problem becomes:

$$x_{\text{avg}} = \int_0^{x_{\text{max}}} x\exp(-x)\,dx \Big/ \int_0^{x_{\text{max}}} \exp(-x)\,dx \ .$$

Because the weight $\exp(-x)$ is larger for smaller x, the x_{avg} will be smaller than half x_{max}.

Accommodating a negative lower limit:

With a non-zero lower limit $x_{\text{min}} = -x_{\text{max}}$ and *upper* limit zero, the problem becomes:

$$x_{\mathrm{avg}} = \int_{-x_{\mathrm{max}}}^{0} x\exp(-x)\,dx \Big/ \int_{-x_{\mathrm{max}}}^{0} \exp(-x)\,dx \ .$$

Because the Boltzmann factor $\exp(-x)$ is largest for the most negative x, the negative x_{avg} will have magnitude greater than half x_{max}.

Accommodating *both* a positive upper limit *and* a negative lower limit together:

With all values from $x_{\mathrm{min}} = -x_{\mathrm{max}}$ up to x_{max} allowed, the problem becomes:

$$x_{\mathrm{avg}} = \int_{-x_{\mathrm{max}}}^{+x_{\mathrm{max}}} x\exp(-x)\,dx \Big/ \int_{-x_{\mathrm{max}}}^{+x_{\mathrm{max}}} \exp(-x)\,dx \ .$$

The x_{avg} will be negative, but not so large in magnitude as half of x_{max}.

About the exponential decline parameter:

For the gravity problem, $x = E / E_{\mathrm{exd}}$, $x_{\mathrm{min}} = -E_{\mathrm{max}} / E_{\mathrm{exd}}$, $x_{\mathrm{max}} = E_{\mathrm{max}} / E_{\mathrm{exd}}$. We do not a priori know E_{exd}. Let us first consider the usual $K_{\mathrm{B}}T$, where K_{B} is Boltzmann's constant, and T is absolute temperature.

The product $K_{\mathrm{B}}T$ first appeared in classical Maxwell-Boltzmann statistics, where it reflects the average kinetic energy of a large number of bouncing, but not bound, particles, such as neutral atoms in a gas. The $K_{\mathrm{B}}T$ then carried over to photons in the blackbody radiation problem. It was vital in removing the Ultraviolet Catastrophe there.

The $K_{\mathrm{B}}T$ later carried over to quantum Bose-Einstein and Fermi-Dirac statistics. The Bose-Einstein statistics apply to photons and other spin-1 particles, which are typically imagined as unbound particles, and so they typically have positive energies. The Fermi-Dirac statistics apply to electrons and other spin-1/2 particles. These particles can be either free or bound, the most familiar bound ones being electrons in atoms. But for electrons bound in atoms, the currently best description is Quantum Mechanics (QM), which is about wave functions and their *total* energy (negative), not just *kinetic* energy (positive). So $K_{\mathrm{B}}T$ is not central in the study of electrons in atoms.

Now let us face the gravity problem. What is the significance of temperature T in that problem? It is not immediately obvious, but let us explore a possibility. For the governing temperature value, consider the temperature known from the microwave background radiation in the Universe, 2.7 degrees Kelvin. Considering how relatively empty the Universe is, that temperature is probably close to the average temperature of the *whole* Universe. The numerical value for K_{B} is $1.38 \times 10^{-23}\,\mathrm{J / deg\ K}$. Therefore:

$$K_{\mathrm{B}}T = 1.38 \times 10^{-23} \times 2.7 = 3.7 \times 10^{-23} \approx 4 \times 10^{-23}\,\mathrm{J} \ .$$

This looks small. But let us compare this $K_{\mathrm{B}}T$ energy to a candidate benchmark for gravitational energy. Consider two charge-neutral, un-excited Hydrogen atoms. The mass of each atom is approximately one atomic mass unit, or one proton mass, $m_{\rm p} = 1.66 \times 10^{-24}$ gm, or 1.66×10^{-27} kg. The gravitational constant is $G = 6.672 \times 10^{-11}$ Newton-meter / kg^2. The radius of each Hydrogen atom is about half an Angstrom, or 0.5×10^{-10} m. So the minimum separation between the mass centers of the two Hydrogen atoms is about $r_{\min} = 10^{-10}$ m. The gravitational potential energy at this $r_{\min}$ separation is about:

$$Gm_{\rm p}^2 \big/ r_{\min} \approx -6.672 \times 10^{-11} \times (1.66 \times 10^{-27})^2 \,/\, 10^{-10}$$

$$\approx -18.385 \times 10^{-55} \approx -1.8385 \times 10^{-54} \approx -2 \times 10^{-54}\,\text{J} \quad .$$

Thus the magnitude of the posited $K_{\rm B}T \approx 4 \times 10^{-23}\,\text{J}$ average energy for a degree of freedom in the Universe is actually *extremely large* compared to the magnitude of the benchmark gravitational energy at minimum separation between the two Hydrogen atoms, $Gm_{\rm p}^2 \big/ r_{\min} \approx -2 \times 10^{-54}\,\text{J}$.

That means the exponential decline parameter $E_{\rm exd}$ must provide *very little* decline; i.e., it must be very *large* compared to $E_{\max}$.

So it is plausible to consider the exponential decline in probability density between $E_{\min}$ and $E_{\max}$ to be nearly linear. Let the symbol ε represent the presently unknown small slope at which the probability density declines with energy.

Then the probability density function $P(E)$ has maximum value $P_{\max}$ at $E = E_{\min} = -E_{\max}$, and it declines from there as:

$$P(E) = P_{\max} - \varepsilon \times (E - E_{\min}) = P_{\max} - \varepsilon \times (E + E_{\max}) \;\; ,$$

And it finishes at $P_{\min} = P_{\max} - \varepsilon \times 2E_{\max}$. Overall, the existence of this decline means there is a slight bias toward negative E .

We can restate the situation in terms of $P_0 = P(E = 0)$. Then the probability density function $P(E)$

$P(E_{\min}) = P_0 + \varepsilon E_{\max}$, and $P(E_{\max}) = P_0 - \varepsilon E_{\max}$, and generally $P(E) = P_0 - \varepsilon E$.

The average energy is then:

$$E_{\rm avg} = \int_{E_{\min}}^{E_{\max}} EP(E)\,dE \bigg/ \int_{E_{\min}}^{E_{\max}} P(E)\,dE$$

$$= \int_{E_{\min}}^{E_{\max}} E\left(P_0 - \varepsilon E\right)dE \Big/ \int_{E_{\min}}^{E_{\max}} \left(P_0 - \varepsilon E\right)dE$$

$$= \left(P_0 E^2 / 2 - \varepsilon E^3 / 3\right)_{-E_{\max}}^{+E_{\max}} \Big/ \left(P_0 E - \varepsilon E^2 / 2\right)_{-E_{\max}}^{+E_{\max}}$$

$$= -\varepsilon E^3 / 3\Big|_{-E_{\max}}^{+E_{\max}} \Big/ P_0 E\Big|_{-E_{\max}}^{+E_{\max}}$$

$$= -\varepsilon (E_{\max})^2 / 3 .$$

We do not know the value of the slope parameter ε, but we do know that $P_{\min}$ cannot be negative, so ε has to have a value such that $\varepsilon \times 2E_{\max} < P_{\max}$; i.e., $\varepsilon < P_{\max} / 2E_{\max}$ .In addition, we can reasonably ask if the value of $E_{\max}$ is actually as important as it looks in the above analysis. This issue is explored next.

A Shaping Function

In some familiar problems from traditional Statistical Mechanics, there is a function creating a gentle enforcement of a physically meaningful boundary. For example, for blackbody thermal radiation, energy has to be non-negative, but we don't have a sharp turn-on at $E = 0$; we have instead a factor of E^2 that rises gently from $E = 0$. It is the ghost of the now-quelled Ultra-Violet Catastrophe phenomenon.

I believe the gravitational problem deserves something like this gentle-rise feature. Furthermore, it deserves its gentle-rise feature not only at $E = E_{\min} = -E_{\max}$, but *also* at $E = E_{\max}$.

The gentle-rise feature can be provided with a function peaked at $E = 0$ and declining to zero, or *near* zero, at both $E = -E_{\max}$ and $E = E_{\max}$. Below are three possible candidates for the gentle-rise factor:

- The product $\left(E - E_{\min}\right)\left(E_{\max} - E\right)$, which provides the mandatory zero values at both boundaries;
- The square of that product, $\left[\left(E - E_{\min}\right)\left(E_{\max} - E\right)\right]^2$, which for either boundary alone resembles what we have in the case of blackbody thermal radiation;
- A Gaussian function centered at $E = 0$; i.e., a function proportional to $\exp(-E^2 / 2\sigma^2)$, where σ is the so-called 'standard deviation'. This kind of exponential function resembles what we see for a classical Maxwell-Boltzmann distribution for a single velocity component in three-dimensional space.

The last of these functions is the one most reminiscent of functions seen in classical Statistical Mechanics, so let us pursue that one.

The classical interaction between two Ampère current elements depends on their current-flow directions in relation to the direction of the line connecting the current elements. Thus the problem overall involves three independent directions. The last of these we can consider more or less steady in time, following just the lumbering motions of macroscopic bodies. But the other two are *very* dynamic, tracking electrons orbiting a nucleus; i.e., tracking dynamically changing neutral Ampère current elements.

Given a particular length of separation, every combination of two neutral Ampère current elements corresponds to a particular value of attractive or repulsive force between those two Ampère current elements interacting at that separation length. The force then corresponds to a particular value of potential energy between neutral Ampère current elements interacting at that separation.

There are huge numbers of Ampère current elements (i.e., atoms) in matter, and hence even more huge numbers of pairs of them. How might we get a handle on this huge problem? Clearly, a statistical approach is needed. We can think about the statistical average of those potential-energy values. We can develop that average as a model for the gravitational attraction, or repulsion, potential between two neutral atoms at the given separation.

Observe that the product of a proposed exponential decline and a proposed Gaussian shaping function can be essentially indistinguishable from an *offset* Gaussian. That is to say, for an appropriate value of ε, the probability density function proportional to two-factors,

$$\exp(-\varepsilon E) \times \exp(-E^2 / 2\sigma^2) \ ,$$

can be essentially indistinguishable from a probability density function proportional to *one* factor:

$$\exp\left[-(E - E_{\text{off}})^2 / 2\sigma^2 \right] \ ,$$

because this exponent expands to:

$$-(E - E_{\text{off}})^2 / 2\sigma^2 = -\left[E^2 - 2E \times E_{\text{off}} + (E_{\text{off}})^2 \right] / 2\sigma^2 \ ,$$

and the last term implies a constant factor of:

$$\exp\left[-(E_{\text{off}})^2 / 2\sigma^2 \right] \ ,$$

which will drop out in the normalization of the probability density function.

Now, how should we represent all possible combinations of two directions in a statistically unbiased way? Integration over familiar spherical coordinates $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$ simply will not do this job. For one thing, the two angles do not have comparable ranges of values. Furthermore, the two angles do not both enter linearly in the integration increment, which is $\sin\theta d\theta d\varphi$. The factor of $\sin\theta$ is needed precisely because sampling θ and φ uniformly would not sample solid angle Ω uniformly.

So instead of working with θ and φ, let us turn to some ancient solid geometry instead. Plato gave us the ideal regular polyhedra; i.e., solid figures that have all faces, all edges, and all corners alike. There exist five regular polyhedra: 1) the tetrahedron (four equilateral triangular faces, six edges, four corners), 2) the cube (six square faces, twelve edges, eight corners), 3) the octahedron (eight equilateral-triangle faces, twelve edges, six corners), 4) the dodecahedron (twelve equilateral-pentagon faces, thirty edges, twenty corners), and 5) the icosahedron (twenty equilateral-triangle faces, thirty edges, twelve corners).

The Platonic regular polyhedra directly provide unbiased ways to sample direction space. We can easily have four, or six, eight, twelve, twenty, or thirty, representative directions normal to the faces, edges, or corners of the regular polyhedra. From those possibilities, we can have 16, 36, 64, 144, or 400, or 900 direction pairs.

And if, in the end, none of these possibilities serves well enough, there remain unlimited opportunities for further discretization beyond the Platonic regular polyhedra. Consider the triangle-faced ones: the tetrahedron, the octahedron, and the icosohedron: each equilateral triangle face can be subdivided into four smaller equilateral triangles: just make one half-sized equilateral triangle and put it upside down at the center of the original full-sized equilateral triangle.

If better resolution is desired, it makes sense to start from the icosahedron. The number of smaller triangles then becomes $20 \times 4 = 80$, and the number of different combinations of two such triangles is then $80^2 = 6400$. For the application of statistical ideas, 6400 is a much larger, and so more satisfactory, number than was the original 400.

Next we could further subdivide, to $20 \times 4^2 = 320$ triangles, yielding $320^2 = 102,400$ combinations of directions. Then we can subdivide to $20 \times 4^3 = 1,280$ triangles, yielding $1,280^2 = 1,638,400$ combinations of directions. This is definitely a huge number. And we can go further; multiplication by *another* factor of 4 is *always* possible. So there are infinitely many possible discretization levels available here.

Note that, in order to make statistical sampling *perfectly* unbiased, we might like all these equilateral triangles to correspond to equal solid angles. But the original figure was an icosahedron, not a sphere, and after subdivision, a slight imperfection emerges. Recall that the original icosahedron had twelve vertices, at each of which *five* triangular faces met. But the smaller triangles then inserted in subdivision meet each other *six* at a time. So when all the triangles are projected from the surface of the icosohedron onto the surface of an enclosing sphere, they are not all exactly similar in corner angles, or in spherical surface areas. (This angle imperfection would, of course, be worse if starting from the tetrahedron or the octahedron. And of course there is a similar issue with square subdivision of the cube.)

But, starting from the icosahedron, the tiny geometric irregularities arising with subdivision can generally be ignored, since we are interested in order-of-magnitude results for understanding gravity a little better, not (yet) in many-decimal-place results for proclaiming total conquest of gravity!

Now, to go along with the discrete sampling of angle combinations, we need a discrete representation of the Gaussian shaping function $\exp(-E^2 / 2\sigma^2)$. Consider a list of discrete binomial expansion coefficients centered at $E = 0$. Let n be the number of polyhedron faces used, and $N = n^2$ be the number of direction combinations used. The binomial coefficients representing the Gaussian are:

$$1, N, \left[N(N-1) \right] \big/ 2, \left[N(N-1)(N-2) \right] \big/ (2 \times 3), etc.$$

Any one of the binomial coefficients has the form:

$$B_I = N! \big/ \left[(N-I)! I! \right] ,$$

for $I = 0$ to $I = N$. The normalization for the sum of the binomial coefficients is:

$$\sum_I B_I = 2^N \ .$$

The list of binomial coefficients makes a number string. Observe that the number string starts at 1, climbs to a maximum, and then declines back to 1. If N is even, the maximum number that occurs in the string is $N! \big/ \left[(N/2)! \right]^2$ and it occurs once, at the middle of the number string. if N is odd (and of course greater than unity), the maximum number that occurs in the string is $N! \big/ \left[(N-1)/2 \right]! \left[(N+1)/2 \right]!$, and it occurs twice, just before and just after, the middle of the number string.

Being discrete, this last example of a gentle-rise function conflicts with the historically continuous vision for the gravity phenomenon. Nevertheless, discreteness does suit any problem for which we might wish to make a computer simulation. That seems to be necessary for many gravitational problems. And discreteness suits any problem for which we might wish to develop a quantum understanding. That kind of understanding for gravity seems to be much sought throughout the physics community. So let us further explore this discrete idea. One really important feature that distinguishes one discretization level from another is the width of the approximately Gaussian peak that each one forms relative to the energy range $-E_{\max}$ to $+E_{\max}$. A finer discretization makes a narrower peak. That is to say, the number of samples grows with N, but the peak width only grows with $n = \sqrt{N}$. A narrower peak reduces the influence of $E_{\max}$. Because of that, we now need to revisit the problem of estimating E_{avg}.For thinking about the gravity problem, a peak offset E_{off} seems a more natural parameter than the exponential decline parameter ε. So consider the formulation

$$E_{\text{avg}} = \int_{-E_{\max}}^{E_{\max}} E \exp\left[-(E - E_{\text{off}})^2 \big/ 2\sigma^2 \right] \Big/ \int_{-E_{\max}}^{E_{\max}} \exp\left[-(E - E_{\text{off}})^2 \big/ 2\sigma^2 \right] .$$

Rendered in discrete form, it is:

$$E_{\text{avg}} = \sum_{I=0}^{N} E \cdot B_I \big/ 2^N \ .$$

It is easy to see that $E_{\text{avg}} \approx E_{\text{off}}$, and that both of them are negative, and that the magnitude of both of them will vary similarly with the level of discretization used. Having zero offset makes E_{avg} zero. Having a tiny negative offset makes E_{avg} a tiny negative number. So there certainly exists an offset that is tiny enough to match gravity. So we have here an existence proof about a possible electromagnetic explanation for gravity. So let us initiate a search for it.

Chasing Gravity

At this point, the reader is equipped with ideas to use in a numerical study. For any such study, the following data will be useful.

Relevant Input Data

Electric permittivity of free space: $\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m;

Magnetic permeability of free space: $\mu_0 = 1.2566 \times 10^{-6}$ H/m;

Speed of light in free space: $c = 1 \big/ \sqrt{\varepsilon_0 \mu_0} = 3 \times 10^8$ m/sec;

Electron mass: 9.1095×10^{-31} kg, Electron charge: $e = 1.6022 \times 10^{-19}$ C;

Hydrogen ground-state orbit radius: $0.529\text{A}^{\text{o}} = 0.529 \times 10^{-10}$ m;

MKS units factor: $4\pi\varepsilon_0 = 4 \times 3.1416 \times 8.8542 \times 10^{-12} = 10^{-10}$ F/m;

Hydrogen atom potential energy: $-e^2 \big/ 4\pi\varepsilon_0 r_1 =$

$$-(1.6 \times 10^{-19})^2 \big/ 10^{-10} \times 0.53 \times 10^{-10}\ \text{J} = -2.6 \times 10^{-38} \big/ 0.53 \times 10^{-20} \approx -4 \times 10^{-18}\ \text{J};$$

Twice electron kinetic energy: $m_e v^2 \approx 4 \times 10^{-18}$;

Squared electron speed $v^2 = m_e v^2 \big/ m_e = 4 \times 10^{-18} \big/ 9.1095 \times 10^{-31} \approx 4 \times 10^{12}$ m^2/sec^2;

Electron speed $v \approx 2 \times 10^6$ m/sec $= (0.0066 \times c)$ m/sec;

Maximum Ampère interaction energy $E_{\max} \approx (v / c)^2 \times$ atom potential energy, or:

$$E_{\max} \approx (0.0066)^2 \times (4 \times 10^{-18})\text{J} = 1.742 \times 10^{-22}\,\text{J} .$$

The Problem to Solve

Observe that the maximum Ampère interaction energy between two neutral Hydrogen atoms, $1.742 \times 10^{-22}\text{J}$, is *much larger* than the corresponding gravitational interaction energy between two neutral Hydrogen atoms, calculated earlier to be:

$$Gm_{\text{p}}^2 \Big/ r_{\min} \approx -6.672 \times 10^{-11} \times (1.66 \times 10^{-27})^2 \,/\, 10^{-10}$$
$$\approx -18.385 \times 10^{-55} \approx -1.8385 \times 10^{-54} \approx -2 \times 10^{-54}\,\text{J}$$

The difference between the maximum Ampère interaction energy, $E_{\max}$, and the corresponding gravitational interaction energy, is about 32 orders of magnitude.

To account for such a difference in terms of the present model, we have to select some candidate discretization levels, and write down the corresponding discrete description of the problem, and show how $E_{\text{avg}} \approx E_{\text{off}}$ can be defined appropriately for that discretization level.

One advantage of a discrete formulation is that, instead of dealing with a continuum of candidate E_{off} values, we can offer a finite number of candidates all related in some simple way to the discretization level. The investigation job may be infinite, but with discretization, it is at least only ***countably*** infinite! Without discretization, one is simply adrift in an uncountable sea of real numbers.

Here are several examples of discretization levels for the gravity problem, each with several related candidate E_{off} values.

Example 1

The most nearly round faces that occur among the Platonic polyhedra are the pentagons on the dodecahedron.

The number of faces is twelve. If we take the twelve vectors from the center of the dodecahedron to its face centers, we will have a set of twelve different, and uniformly distributed, direction vectors.

The number of combinations of two such directions is then $12 \times 12 = 144$. The binomial expansion coefficients for this situation are:

1, K 144, K (144 x 143) / 2, K (144 x 143 x 142) / (2 x 3), …
…144! / $(72!)^2$, K 144! / $(72!)^2$, …
(144 x 143 x 142) / (2 x 3),(144 x 143) / 2, K144, K1, K …

Except for the first few and the last few, these binomial coefficients as written are tedious to evaluate because of the factorials involved. But for troublesome factorials, we can always use Stirling's approximation [5]:

$$\ln(Q!) \approx Q\left[\ln(Q) - 1\right].$$

The normalization for all these binomial coefficients is

$$2^{144} \approx 2.2301 \times 10^{43} \ .$$

This is a big number, well surpassing the 32 orders of magnitude sought to model gravity.

Now consider the average value of E. If the energy distribution is centered on $E_{\mathrm{ctr}} = 0$, then it is certain that $E_{\mathrm{avg}} = 0$.

If the energy distribution is centered on a finite negative E_{ctr}, then E_{avg} must be negative too. But it may be slightly different than E_{ctr}. For example, $E_{\mathrm{ctr}} = -E_{\max} / 144$, then E_{avg} is also *almost* one increment back, at $-E_{\max} / 144$. But it is not quite a whole increment back, because, while the lower limit for energy remains $E_{\min}$, the upper limit becomes $E_{\max} - E_{\max} / 144$. As a result, the set of binomial expansion coefficients looses one member, and starts at 144 instead of 1.Neglecting such small adjustments, let us look first at this candidate offset of $E_{\mathrm{off}} = -E_{\max} / 144$. This candidate offset is not nearly small enough in magnitude to represent gravity. It is hopeless for this job.

Next, let us look at an offset of $E_{\mathrm{off}} = -E_{\max} / 2^{144} = -E_{\max} / 2.2301 \times 10^{43}$. This candidate offset is smaller in magnitude than needed to represent gravity, by about ten orders of magnitude. But in response we could let the gravitational offset be more than one unit. So in principle this candidate is viable. Finally, let us look at an offset $E_{\mathrm{off}} = -E_{\max} \Big/ \sqrt{144 \times 2^{144}} = -E_{\max} \Big/ 12 \times 2^{72} = -E_{\max} \Big/ 5.6668 \times 10^{22}$. This candidate offset is larger in magnitude than needed to represent gravity, by about ten orders of magnitude, so this one is hopeless. In summary, this Example offers one candidate offset E_{off} small enough in magnitude to invite further study.

Example 2

The largest number of faces that occurs among the Platonic polyhedra is twenty. If we take the twenty vectors pointing from the center of icosohedron to the centers of its twenty faces, then we will have a set of twenty different, and uniformly distributed, direction vectors.

The number of combinations of two such directions is then $20 \times 20 = 400$. The binomial coefficients for this situation are:

$$1, 400, (400 \times 399) / 2, (400 \times 399 \times 398) / (2 \times 3)...$$
$$...400! / (200!)^2, 400! / (200!)^2 ...$$
$$...(400 \times 399 \times 398) / (2 \times 3), (400 \times 399) / 2, 400, 1$$

The normalization for all these binomial coefficients is:

$$2^{400} \approx 2.582 \times 10^{120} \quad .$$

This is a number much larger than the corresponding number from Example 1, which was $2^{144} \approx 2.2301 \times 10^{43}$.

Now look at the approximate $E_{\text{off}} = -E_{\max} / 400$. It is not nearly small enough in magnitude to relate to gravity.

Next, let us look at $E_{\text{off}} = -E_{\max} / 2^{400} = -E_{\max} / 2.5822 \times 10^{120}$. This is much smaller in magnitude than needed, but we can look at multiple increments of this size.

Finally, let us look at

$$E_{\text{off}} = -E_{\max} \Big/ \sqrt{400 \times 2^{400}} = -E_{\max} \Big/ 20 \times 2^{200}$$
$$= -E_{\max} \Big/ 20 \times 1.6069 \times 10^{60} = -E_{\max} \Big/ 3.2138 \times 10^{61} \quad .$$

This candidate offset is also smaller in magnitude than needed to represent gravity, but again we could consider multiple units of this size.

In summary, this example offers two candidate offsets E_{off} small enough in magnitude to invite further study.

Also, recall that the root number 20 invites indefinitely much further study because it invites indefinitely many subdivisions of its triangle faces, producing indefinitely many further candidate offsets to study.

Example 3

The largest number that occurs anywhere in the descriptions of Platonic polyhedra is thirty. If we take the thirty vectors pointing from the center of the dodecahedron to the centers of its thirty edges, or, equivalently, from the center of the icosahedron to the centers of its thirty edges, then we will have a set of thirty different, and uniformly-distributed, direction vectors.

The number of combinations of two such directions is then $30 \times 30 = 900$.The binomial coefficients for this situation are:

$$1, 900, (900 \times 899) / 2, (900 \times 899 \times 898) / (2 \times 3) \ldots$$
$$\ldots 900! / (450!)^2, 900! / (450!)^2 \ldots$$
$$\ldots (900 \times 899 \times 898) / (2 \times 3), (900 \times 899) / 2, 900, 1$$

The normalization for all these binomial coefficients is 2^{900}.To evaluate this, use: $2^{300} = 1.9424 \times 10^{130}$, making

$$2^{900} \approx 7.3285 \times 10^{390} \, .$$

Now look at the approximate $E_{\text{off}} = -E_{\max} / 900$.It is not nearly small enough in magnitude to relate to gravity.

Next, let us look at $E_{\text{off}} = -E_{\max} / 2^{900} = -E_{\max} / 7.3285 \times 10^{390}$.By itself, it is way too small in magnitude to speak to gravity.But as always, we can look at multiple units.

Finally, let us look at

$$\begin{aligned} E_{\text{off}} &= -E_{\max} \Big/ \sqrt{900 \times 2^{900}} = -E_{\max} \Big/ 30 \times 2^{450} \\ &= -E_{\max} \Big/ 30 \times 2.7071 \times 10^{195} = -E_{\max} \Big/ 8.1213 \times 10^{196} \end{aligned}$$

This candidate offset is also smaller in magnitude than needed to represent gravity, but again we could consider multiple units of this size.

In summary, this example also offers two candidate offsets E_{off} small enough in magnitude to invite further study.

Conclusion

This Chapter breaks ground, but does not pretend to finish the needed excavation, much less build the desired edifice, concerning gravity. Its purpose is just to persuade the reader that gravity may not be a thing apart from electromagnetism, and it also may not be a thing apart from QM. Instead, gravity may be understandable as a statistical residue from a huge number of electromagnetic interactions.

The interactions themselves are presumed to be sometimes attractive, but sometimes repulsive, averaging out to just very slightly attractive. The detailed model for them predates Maxwell, arising from the work of J.M. Ampère concerning chare-neutral current elements. The Ampère approach has a lot of specific geometric detail, all of which is necessary to produce specific vector forces on specific current elements. But, for a statistical view, we are more interested in energies than in forces. So, the present work does not deal with the forces at all, and speaks only of energy.

That change of focus somewhat mimics the change of focus that comes with changing from the Newtonian view of gravity to the Einstein view of gravity. Newton dealt with instantaneous forces between specific masses, driving their trajectories. Einstein dealt with the metric tensor, which belongs to unified four-dimensional space-time, which is created by resident masses, which by $E = mc^2$ are equivalent to energies.

It is worth noting that any statistical view of gravity is, by definition, somewhat similar to the prevailing statistical view of atoms; i.e., the view offered by QM. Such a statistical view is not about individual particles/planets and their individual trajectories in time. So it is not about the mechanics of interaction: signals, signal speed, etc. So while the present approach follows GRT in its emphasis on energy, it avoids SRT in its avoidance of specific trajectories driven by c -speed signals.

The subject of gravitational signals, and the resulting evolution of galaxies, and the Universe as a whole, is another domain of investigation. But, again, the subject ends up being most easily treated statistically, with emphasis on potential energy. Both the disc shape of so many galaxies, and their typical barred-spiral structure, are understandable in terms of a driving super-massive two-body system at the center of a large population of ordinary stars, each of which has negligible massive compared to the driving two-body system.

For the interested reader, this subject is treated in [6].The stellar trajectories turn out to look like the outlines of two-humped ratchet wheels. The in-fall segments correspond to the highest density of stars, and so make the conspicuous spiral arms. The out-walk segments illustrate an important point: when signal propagation speed is finite, and system evolution involves orbits, it is possible for signal communication to get slightly out of phase, with the result that attraction becomes repulsion. Disc galaxies spread out. The Universe expands. So it goes.

Acknowledgments

The Chapter is dedicated to the memory of the late Peter Graneau, courageous experimenter, valued colleague, and friend.

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