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Chapter 1

CHAOS THEORY AND FINANCIAL STATEMENTS

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ABSTRACT

The purpose of this chapter is to show some applications of chaos theory to financial statements. Although chaos theory has been used in several financial topics, it is not usually involved with financial statements, which seem to remain impervious to it. To describe the analytical possibilities that chaos theory has in financial statements, a description of some features of chaos theory and their interest with regard to financial statements is made. Next, the application of chaos theory in a specific topic, such as the analysis of the accounting equation is presented. In addition, explanations of the possibilities of chaos theory in analyzing financial statements accounts and ratios, using several models of chaos theory, are provided. Finally the use of recurrence analysis is described to provide different perspectives on financial statements information. The intention is to create an interest in viewing financial statements as a complex information system.

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1. INTRODUCTION

This chapter introduces some insights into the applications of chaos theories in financial statements analysis. To this purpose, the chapter comprises analyses and principles and rationales that justify the potential of using chaos theory applications in financial statements. The intention is to provide some directions and suggestions in a field that is nearly nonexistent. Financial statements are the core of business; they are where companies show their strengths and weakness, and also the information tool for the management team to make decisions.

The mathematical and logical explanation introduced in this chapter give both an intuitive and a formal comprehension of the chaos phenomenon in financial data, mainly from a deterministic point of view, but also with some subjective rules and interpretations; so both quantitative and qualitative views are relevant.

Financial statements apparently have a fixed structure and the reporting rules of financial information forces analysts to operate in a rather normative way. However, financial statements are far from being as deterministic as they look. Several other analytical tools used in financial statements are not so immovable; for instance, the Management Discussion and Analysis and Notes to Financial Statements sections are valuable tools for the manager to analyze data and make decisions. These tools are usually not expressed in a numerical but a verbal language. Besides, even normative operations admit a subjective interpretation.

Within this framework, many difficulties exist when trying to fit chaos models and theory to financial statements. Financial data, in the usual spreadsheet arrangement of financial statements, do not have a proper structure for analysis with the tools of chaos theory. Besides, the vertical orientation of many financial statements analysis does not allow any easy chaos explanation; furthermore, when looking for longitudinal data, they are not long enough to support a chaos trend or recurrence analysis. In this regard, new analytical-rational deductive and simulation models along with new ideas are required in this field. Some techniques that may be helpful for fitting chaos theory to financial statements are described in the chapter; the intention is to advance the comprehension of the application of chaos theory to financial statements, as a result of previous research and the scarce literature about the topic. Therefore, the purpose of this chapter is to provide some insights into the use of chaos theory and models in financial statements.

The chapter starts with a description of the nature of chaos, pointing out some distinctive characteristics. There follows a reflection on the interest that chaos theory has to financial statements. Next, the view of financial statements as a chaotic system is introduced and divided into three sections. The first section discusses an alternative approach to the accounting equation and presents several analytical alternatives including the stochastic approach and the catastrophe and chaos theories. The second section focuses on the analysis of data series, as this is an important flaw in financial statements. Finally the last section introduces recurrence analysis as an important tool in financial statements to detect chaotic patterns. Throughout this chapter, the message is that financial statements are more than a static structure: they are a complex conceptual and analytical system.

2. THE NATURE OF CHAOS

Sinai (2010) mentions two relevant chaos definitions: one of them is the notion that chaos is associated with the appearance of periodic orbits of all periods, which was proposed by Li and Yorke (1975); the other, and much more popular notion of chaos, is the sensitive dependence on initial conditions (Lorenz, 1995, p. 8).

The notion of chaos implies sensitive dependence on initial conditions, topological transitivity and denseness of periodic points (Degirmenci and Kocak, 2010), which is the classical Devaney definition (1989). In this regard, it is a topological space with a metric *d* (a metric space) where every point shows a recurrence; i.e., for certain n: $f^{n}(x) = x$.

The nature of a chaotic set relies on a large number of unstable periodic orbits and sensitive dependence on minimal perturbations (Pyragas, 2006); moreover, any change in determining the initial conditions and also the uncertainty about these conditions makes it impossible to determine the trajectories beyond a certain horizon (Pontes, 2016). Thus, sensitivity to initial conditions and instability are characteristics of chaos. A small change in initial conditions has exponential growth resulting in deterministic chaos (Gauthier, 2009).

Therefore, a chaos system exhibits sensitive dependence, determinism and nonlinearity (Smith, 2007, p. 1), and the most salient characteristic of chaos is the unpredictability of the system's behavior (Gu and Chen, 2014).

Sometimes, chaos systems are taken as a part of complex adaptive systems. In this regard, a complex adaptive system exhibits characteristics such as multilevel, multidisciplinary, system evolution patterns, attractors, chaotic structures, observer-dependent description, emerging control and order, fast transition between equilibrium points, adaptation and self-organization, and processes such as autopoiesis, dynamic and dissipative systems, as well as the dynamics of chaos (Dooley, 1997).

On the other hand, chaos systems, according to Dooley (1997), exhibit chaotic changes, scale and incremental-cumulative changes alternating between inertia and change, punctuated equilibrium, real or perceived change or stagnation, and movements away from equilibrium. Therefore, taking into consideration the number and type of characteristics that both systems have, it is possible that chaos is a part of the complexity theory.

Complexity theory describes the responses of people to chaos, which does not mean being out of order, but emerging order with no predictability (Faggini and Parziale, 2016). Moreover, chaotic behavior is relevant in complex systems (e.g., Anderson, 1999; Dooley, 1997; Dooley and Van de Ven, 1999), and is closely linked to complexity theory (Mouck, 1998) or is a part of complexity theory (Klijn, 2008). In addition, it is identified as belonging to the second generation of complexity (Alhadeff-Jones, 2008). Nevertheless, some issues arise when taking chaos as a synonym of complexity, because it would not make any sense to talk about complexity and chaos as if they were the same, or as if chaos were just a subsystem of complexity.

Deterministic and mathematical chaos explain the acquisition of complex characteristics by simple models (Morel and Ramanujam, 1999) and the dynamics of chaos is essential to assert the existence of complex adaptive systems (Dooley, 1997). On the other hand, it has been pointed out that complex systems and chaos are not related to each other (Cilliers, 2000). Thus, it might be that complex systems are not chaotic and chaotic systems are not necessarily complex.

Chaos theory might also have a connection to catastrophe theory (McKelvey, 1999). Catastrophe theory explains how some continuous causes lead to discontinuous effects, it is a sudden transition to a new but not necessarily worse state (Jakimowicz, 2010), with discontinuity, divergence, multimodality, alternativity and inaccessibility (Jakimowicz, 2010). Both catastrophe and chaos theories rely on bifurcation and exhibit topological and phase space changes when changing the control parameters (Gu and Chen, 2014). Therefore, some morphological similarities exist; however, the catastrophe phenomenon implies a sudden change with no return to a previous state, while chaos implies a recurrence.

3. The Interest of Chaos Theory to Financial Statements

The science of chaos is used in different business fields, such as in supply chains (Ramírez and Peña, 2011), governance problems (Gangopadhyay, Elkanj, and Rahman, 2011), models of firm profit (Lenka, 2009), business cycle (Jakimowicz, 2010), agriculture extension (Bordbar, Malekmohammadi, Hosseini, and Chizari, 2015), marketing research (Diamond, 1993), management planning (Frear, 2011), and many other topics. It has attracted researcher interest, particularly in finances; e.g., in financial crisis (Haley, 2010; Nieto, 2009) and markets (Bastos, 2013; Guhathakurta, Bhattacharya, Banerjee, and Bhattacharya, 2013; Jinguang, and Li, 2008; Spronk, and Trinidad, 2005), nonlinear finance systems (Kocamaz, Göksu, Taşkın, and Uyaroğlu, 2015), and finance and power (James, 2014), among many others.

Although chaos theory is useful for research in business and finance (Mouck, 1998), its applications in financial statements are scarce; financial statements seem to be impervious to variability and uncertainty and, consequently, they do not need a chaos approach. They have a fixed structure and a high normativism in financial information reporting. Although the Management Discussion and Analysis and Notes to Financial Statements sections are essential non-numeric tools for the manager in explaining data and making decisions, the main core of financial statements is expressed in numerical code, which could favor a deterministic chaos approach.

Nevertheless, many difficulties exist when trying to fit chaos models and theory to financial statements. The financial data spreadsheet seems not to be appropriate to the usual analysis of dynamical systems and chaos tools. The annual, monthly or even weekly financial statements report is a snapshot of all the financial events that happened during that period, but they do not reach the long data series needed for chaos trend analysis. There is also a vertical orientation in many of the financial analyses typically performed.

As indicated in economics, there are only small data samples and the literature still does not provide solid support for the use of chaos theories (Faggini and Parziale, 2016). Hence, new models must be incorporated to test the existence of chaos in financial statements; in this task, analytical-rational deductive, empirical and simulation models along with new analytical ideas are required.

4. Financial Statements as a Chaotic System

4.1. An Alternative Approach to the Accounting Equation

Regardless of the appearance of immobility of financial statements, their principles have been analyzed, and sometimes criticized, suggesting new approaches. Several authors proposed logical paradigms to explain the basis of financial statements, resulting in extremely sophisticated axiomatic systems that deal with some parts of, or the entire, accounting operation (Carlson and Lamb. 1981; Ijiri, 1986, 1989; Mattessich, 1964; Willet, 1987, 1988, 1991).

One of the foundations of financial statements is the assets–claims on assets equality, which results in the accounting equation $A = L + S$. There are some aspects of this equation and its logical basis that deserve attention. These aspects are related to the duality principle, or the property that monetary units have of being assets and claims on assets simultaneously (Juárez, 2016a, 2016b, 2016c), which suggests that the accounting equation has non-classical logic properties (Juárez, 2013, 2014a). A way of reasoning including contraries is implicit in financial statements, indicating the existence of many logics, e.g., the existence of dialogic, which can deal with contraries (Juárez, 2013, 2014a). In this context, an accounting transaction has an effect on assets *a*, liabilities *b*, or stockholders' equity, such as $a \wedge (l \vee s)$; i.e., a conjunction exists among these terms of the equation, giving to the proposition "accounting transaction has an effect on" a true–false value.

The problem is that an accounting transaction can involve different accounts with opposite properties; therefore, having a transactional effect on assets and also on liabilities (claims on assets) accounts means *a* (assets) = *l* (claims on assets). In that sense, something is owned (*a*) and, at the same time, it is recognized as a debt (*b*), because the same quantity is located in those two accounts. Every monetary unit has this property and, accordingly, assets and claims on assets sets are considered equal in the balance sheet; the total amount of monetary units in assets is equal to the total amount of monetary units in claims on assets.

However, it has been demonstrated that the balance sheet comprises a transitive hierarchy of sets $I_{iLn} \subseteq I_{iLn-1}, \ldots, \subseteq I_{iL3}, \subseteq I_{iL2}, \subseteq I_{iL1}$, ($I_{iLn}$ the lowest and I_{iL1} the highest level accounts), so having assets and claims on assets organized in this manner and using the Zemerlo–Fraenkel set theory axioms, the analysis led to the conclusion that assets and claims on assets are not equal (Juárez, 2015, 2016a). In addition, it has been demonstrated that their cardinality is not the same, so they are not equivalent (Juárez, 2016b) and, finally that their total amount of monetary units is not the same (Juárez, 2016c).

When the analysis includes the duality principle, the accounting equation results in (Juárez, 2016d)

$$\sum_{h=1}^{k} \sum_{s=1}^{t} u_{ahs} = \sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{chij} \tag{1}$$

with u_{ahs} = assets monetary units; u_{chij} = claims on assets monetary units; h = sequence number for assets accounts A_i; s = sequence number for the monetary units in A_i; i = sequence number for claims on assets accounts C_i; j = sequence number of monetary units in a C_i, images of the monetary units in an A_i; $\exists m$ = a set of monetary units in a C_i exists and is found, such as they are images of some monetary units of an A_i,; and $\exists n$ = a C_i exists, such as it has images of some monetary units of an A_i, for every domain A_i.

It is a recursive procedure that runs until all A_i and C_i are scanned, and all the ranges are found, according to the dual principle. The procedure can also be run in the opposite direction resulting in (Juárez, 2016c),

$$\sum_{h=1}^{k} \sum_{s=1}^{t} u_{chs} = \sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{ahij} \tag{2}$$

The existence of these two similar equations is relevant, but not to be analyzed here; for the purpose of explaining the implications that these equations have regarding chaos, only the first one (1) will be taken. Therefore, as it is described (Juárez, 2016c, 2016d) the three-dimension system on the equation's right side is transformed into a two-dimension system by a coordinate transformation, in the form: $c_x^2 = s_x c_x^3 + c_x$; $c_y^2 = s_z c_z^3 + c_z$, with

c_x^2, c_y^2 the coordinates in the new two-dimension system, and c_x^3, c_y^3 the coordinates in the three-dimension system.

Every monetary unit, in this case of claims on assets, is transformed by these linear equations and the coefficients s, c are parameters of the value change of monetary units, slope and constant, respectively. Therefore, the problem is to find an appropriate solution to obtain all of these coefficients and terms in the transformation equations.

The solution admits several approaches; some of them are as follows: a) a stochastic solution with a probability density and a kernel operator; b) a cusp catastrophe model; c) a chaotic model. For the purpose of the analysis, let us assume all of these solutions lead to claims on assets–assets equality; however, it must be understood that this could not be so.

The solution a) uses a probability density function with a kernel operator and the mathematical expression was exposed in a previous paper (see Juárez, 2016d); it is as follows:

$$E(C) = p_1 \int_{-\infty}^{\infty} k(p,l)\ c_{zx}(l)dl + p_2 u_{chij} \int_{-\infty}^{\infty} k(p,l)\ s_{zy}(l)dl \quad (3)$$

where $k(p, \cdot)$: $(-\infty, \infty) \rightarrow \mathbb{R}$; $k(p, l)\Delta l$ is the probability of having a value c_{zx}, s_{zy} between l and Δl and $k(p, l) \geq 0$. In this solution, an appropriate kernel must be found, such as the Dirac or uniform kernel, among others (for a description of types of kernels see Neamt, 2014).

Approach b) introduces a cusp model of catastrophe theory, the formulation is

$$V(y) = y^4 + by^2 + ay \quad (4)$$

the stability condition of which is (Barunik and Vosvrda, 2009)

$$\frac{dV}{dy};\ 0 = y^3 + by^2 + a \quad (5)$$

with a, b control parameters of the system. These authors mention the parameters λ and σ (location and scale parameters respectively) introduced by Cobb (1980); introducing these parameters, the solution is (see Cobb, 1980)

$$0 = a + b\ \frac{(y-\lambda)}{\sigma} - \left(\frac{(y-\lambda)}{\sigma}\right)^3 \quad (6)$$

or, in cubic form

$$0 = \alpha + \beta z - z^3 \tag{7}$$

In both of the previous cusp models, with or without location and scale parameters, the control parameters a and b depend on the independent variables; in the accounting equation, as previously introduced, they are c_{zx}, s_{zy}; therefore, their influence is

$$a = \sum_{i=1}^{n} a_i \, \mathrm{c}_{(zx)i} \ ; \quad b = \sum_{i=1}^{n} b_i \, \mathrm{s}_{(zy)i} \tag{8}$$

As a result of the previous transformation of monetary units in financial statements, parameters c_{zx}^2, c_{zy}^2 change the value of the monetary units; however, in order to keep the total claims on assets equal to total assets, an increase in the value of one monetary unit leads to a decrease in the value of another monetary unit.

In the two-dimension system, the value sequence of monetary units is still unknown; however, despite that, if total assets is taken as a boundary once claims on assets amount to a certain value it cannot increase again. So there is a point in which the new values of claims on assets must go in opposition to the boundary reached.

The stochastic model as used by Barunik and Vosvrda (2009) is

$$dy = \frac{dV}{dy} dl + \sigma_y \, dW_l \tag{9}$$

In the case of its application to claims on assets monetary units, *dl* is the differential of the location of monetary units in claims on assets accounts in the two-dimension system, σ_y is the variance of new monetary unit values, and W_l is a Wiener process ($dW_l \sim N(0, dl)$).

In this manner, a stochastic procedure as described by Cobb (1980) is used to apply the cusp model in the transformation of claims on assets monetary units and, thus, in the relationship between assets and claims on assets.

Approach c) introduces a chaos model to generate a new two-dimension map with new coefficients (c_{zx}, s_{zy}). Therefore, this leads to uncertainty in the results; the model takes the transformation equations of a Hénon map as (Alligood, Sauer and Yorke, 1996, p. 51)

$$f(x, y) = (-a + x^2 + by, x) \tag{10}$$

Furthermore, in a data series with a delay form, it is $x_{n+1} = 1 - ax_n^2 + by_n$, $y_{n+1} = x_n$. Thus, the resulting map is based on the parameters *a*, *b*, which usually are *a* = 1.4, *b* = 0.3.

These parameters can have some variations and result in a chaotic map. Therefore, knowing the condition of total claims on assets being equal to total assets, given the parameters *a* and *b*, and based on the definition by Alligood et al. (1996, p. 58), an attractor is a fixed point *p* where $f(i) = i$, with $\varepsilon > 0$, for all $j \in (p-\varepsilon, p+\varepsilon)$ such as $\lim_{k\to\infty} f^k(j) = p$, *p* being total claims on assets equal to total assets. The attractor is a sink and an unstable point. This method gives the possibility of creating the whole map with the assistance of predefined parameters.

It is also possible to obtain a stochastic approach by introducing a random noise or creating a series of functions, such as several initial states leading to a probabilistic approach.

4.2. A Perspective Based on Sector Data

The previous section's approach was an analytical–rational–deductive one, but the analysis can take a more empiric development. To use a larger number of data the analysis can take sector financial statements or even all the financial statements that companies report to government institutions. These data have a link between them in their orderly values and are a sample that captures the distribution of financial statements data in the whole sector or economy. The connection among data has support in a logical reasoning regarding the ordering that the analysis requires; this reasoning can use a logical increment in the profits of the companies or the economy, or a specific relationship between financial statements ratios, among others. Besides, determining the distribution and performing a sampling procedure can give more support to the analysis validity.

A first use of this approach was in the crude oil extraction and natural gas sector; Lorenz equations were used with cash flow, profit and loss, and total assets accounts as input data, to obtain a good predictive model of the relationship among them (Juárez, 2010a). The equations were used in the following manner

$$x\left(\frac{dx}{dt}\right) = 10((l_n(cash\,flow)) - (l_n(profit\ and\ loss))) \tag{11}$$

$$y\left(\frac{dy}{dt}\right) = (l_n(profit\ and\ loss))(0 - (l_n(assets))) - (l_n(cash\,flow)) \tag{12}$$

$$z\left(\frac{dz}{dt}\right) = (l_n(profit\ and\ loss))(l_n(cash\,flow)) - \left(\frac{8}{3}\right)(l_n(assets)) \tag{13}$$

Their complex nature comes from the specific relationships created among them by these computations; they are not in the usual linear manner of independent variable and dependent variables, which show linear independence and relationships. That intuitive application resulted in a satisfactory prediction with 73% explained variance and offered an approach to use chaos equations in an easy way to improve the prediction among financial indicators. Nevertheless, the obtained relationship acquires a more complicated nature, and explanation becomes more difficult.

Another similar application with the same model was in the health sector, where using the computations, again with Lorenz equations and cash flow, profit and loss, and total assets as input data, resulted in predictions of 80% and 96% of explained variance (Juárez, 2010b).

These applications of a chaos model had the intention of reaching a better explanation of the relationships among financial indicators and not to model some data as chaotic. They employed a method to make linear predictions but using a transformation resulting from the chaos theory and were supported by an intuitive identification of the chaos phenomenon by looking at the data graphs.

In other reports (see Juárez, 2012, 2013), several chaotic effects on financial statements data appeared. Taking the difference, with a delay of one, in the total assets data series $a_x - a_{x-1}$, produces a turbulence followed by a sudden increase in total assets values. In the turbulence, the existence of chaos was suggested by the distribution of frequencies in several groups in a Fourier analysis, such as is proposed by Brown (1995, p. 35). Furthermore, a quadratic equation was used to explain the profit and loss data evolution, showing up a quadratic map effect that may be invisible in the ordinary way of depicting data. The equation was $y = \mathrm{f}(x) = x^2 - 5x$ and resulted in a parabola, with a bifurcation, where zero is an unstable point, and minimal moves make profit and loss move away from zero through different paths.

Another effect is observed when looking at stockholders' equity or cash flow at end of year, both of them indexed by profit and loss; different profit and loss values are associated with the same values of cash flow at end of year and stockholders' equity, suggesting the nonexistence of a unique solution to explain this relationship and the difficulty in prediction making. Among the obtained effects, the analysis found how minimum changes in one of those accounts resulted in dramatic changes in profit and loss, quickly alternating between the two and making a linear interpretation impossible. Different values of these other items are associated to profit and loss, grouped around some points, and after deviations they return to that area. Therefore, small differences in profit and loss produce large changes in stockholders' equity and cash flow at end of year, and the other way around is also true. Therefore, some points are attractors to the function values.

All of these findings are indicative suggestive and introduce the language of chaos to the management view of financial statements; however, they offer an entirely different perspective on financial statements and new analytical possibilities. Nevertheless, they also require the confirmation of the existence of chaos from a mathematical viewpoint, and with the same rigorous method as those proposed in the previous section.

4.3. Recurrence Analysis Approach

Recurrence analysis is also used in financial statements data analysis. This analysis is intended to identify chaotic patterns and attractors in data series, and it combines statistical analysis with an attracting and suggestive graphical presentation of phase spaces. Phase spaces contain all states of a system; it is the system's evolution over time (Guhathakurta, Banerjee and Dan, 2013).

The basis of its application to financial statements was described in Juárez (2014b) and Juárez, Mesa, and Farfán (2014); as described in those papers, data are a function of time $Y_t = F(Y_{t-n})$, with a delay t-n, and a phase space plot $X_t = F(X_{t+delay})$, where every point is plotted against every other point with a delay $y(t\text{-}d)$ (or advancement $y(t+d)$). Every delay creates a dimension m_i, which leads to an m-dimension space $\mathbb{R}^m$ that embeds the original series.

Every delay d and number of dimensions m originate a vector $v(i) = \{x_i, x_i\text{-}d, x_i\text{-}2d, \ldots, x_i\text{-}(m\text{-}1)d)\}$ and, consequently a family of vectors $V = \{v(1), v(2), v(3), \ldots, v(N\text{-}(m\text{-}1)d)\}$. A norm is defined to represent data in a matrix M, in such a way that for every point pair, their value is 1 if their distance is less than a given threshold, otherwise it is 0; i.e., $M=\{\forall m_{ij} \mid m_{ij} = 0$ if $|v_i - v_i| > \varepsilon; m_{ij}$

= 1 if $|v_i - v_i| \leq \varepsilon$}. An *m*-dimensional space has the form {$S(t)$, $S(t-d)$, $S(t-2d)$, . . . , $S(t-(m-1)d)$} and every point $y(t)$ is plotted in coordinates {$S(t)$, $S(t-xd)$)}, $1 \leq x \leq m$-1.

Therefore a variable data series is embedded in an *m*-dimension space. However there must be a logic to order the values of the variable, and, as mentioned earlier in this chapter, if the variable follows a certain order, then it might be that another variable is influencing that order; i.e., one or several variables have to influence the values of another variable, so the latter has its values ordered by the values of the former, that is $(x_1 \ \& \ x_2 \ldots \ \& \ x_n) \rightarrow y$; it has been said this sample of data, when ordered by another variable, reflects the order of that variable in the population (Juárez, 2014b). Under this arrangement, the analysis can obtain a phase space of the variable *y*.

Some measures in the analysis are: determinism or predictability in the system, percentage of recurrence, laminarity or the number of recurrence points, entropy or equiprobability of events and trapping time or mean time in a specific state.

Using this method and the financial rationality regarding the relationship between variables, the impact of monetary policy on net cash flow from investment-operating and liquidity was analyzed in financial statements (see Juárez et al., 2013), showing the chaotic patterns that emerge in the data series of net cash flow from investment activities, ordered by total assets, and with a lag of $\tau = 3$, and 2 embedding dimension. The dependence between these variables was shown to be nonlinear. Predictability of the system was 34.26% with a recurrence of 51.99%, among other fit measures. In that research the same method was used to test the relationship between total assets, working capital and net cash flow from operating activities, with similar results. Thus, the utility of recurrence analysis in showing patterns that are not easy to see with simple graphs and analysis was proved. The findings of this research is that monetary policy influence acts on a complex structure, resulting in high unpredictability and creating turbulence instead of a clear direction for companies.

Another research, using the same method, explored the relationship between fixed assets, infrastructure and financial health in the hospitality industry (Juárez, 2014b). However, on that occasion, a data transformation was conducted to improve the predictability of results. A Hénon map was created by using $f(x, y) = (a - x^2 + by, x)$; for instance, in the case of return on fixed assets (l_R) and fixed assets net worth (l_N), it was $X_1 = (1.28 - (l_R)^2) - (0.01\ l_N)$ and $Y_1 = l_R$.

Using this transformation, a large trend could be seen for data to go back to previous states and stay there. In addition, the degree of accomplishment in infrastructure by the government did not prove to create a more linear dependency on the company performance either. Thus, according to the results, the financial health of the hospitality industry is not associated to investment in fixed assets and infrastructure in a linear and monotonic manner; also, more investment, better development of infrastructure, and an increase in fixed assets value do not lead to improving the financial health of companies. Nonlinearity and complex relationships are a norm in the results; therefore recurrence analysis goes further than a simplistic way of interpreting financial data.

In using this method in financial statements, precautions need to be taken, as only one variable is the subject of recurrence analysis. The variable must have its values in a sequential manner; i.e., where time is not the index to the values, then another variable or set of variables must be used, provided that they fit the logical requirements of the topics under scrutiny and the analytical demands of a series data. Furthermore, a statistical sample of data can be included in the analysis or even bootstrapping can help in obtaining more accurate results and a better statistical justification.

CONCLUSION

The use of nonlinear dynamical systems and sophisticated chaos theory analysis are still to be developed in financial statements analysis. Some ideas have been provided in this chapter, most of them resulting from previous efforts to introduce this perspective mainly in financial accounting and corporate finances. Nevertheless, without any doubt financial statements, under the view of chaos theory, appear to be a more complex structure than their simple presentation on a spreadsheet, and give the impression of admitting very sophisticated analysis and formulations.

The usual ratio, vertical and horizontal analysis of financial statements, and even more complicated analytical forms, such as those using statistical models, can be flawed in the way they understand the financial reality and also the financial statements principles. In this regard, some efforts have been made resulting in the suggestion that this important informative core of data for companies must evolve towards a different comprehensive system. This chapter contributes to those efforts.

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